University of Pittsburgh

CS 1502 Spring 2009 Exam 1

There are a total of 100 points. This is a closed book exam.

We can't answer questions like *What do you want for this question?* or *I don't understand the question.* That would be too disruptive, and wouldn't be fair, because some people would get more information than others.

Follow the instructions, and use your best judgment.

Good luck!

1. (12 points) Translate the following English sentences into first-order logic.

   (a) Neither **a** nor **b** is the same size as **c** and in front of **d**.

   ```
   neither a nor b is BLAH
   ~(BLAH(a) v BLAH(b));  BLAH = samesize(*,c) ^ frontof(*,d)
   ~((samesize(a,c) ^ frontof(a,d)) v (samesize(b,c) ^ frontof(b,d)))

   ~((P ^ Q) v (R ^ S)) equiv
   ~(P ^ Q) ^ ~(R ^ S)  equiv
   (~P v ~Q) ^ (~R v ~S)
   ```

   (b) If **a** is in front of **b**, then if it isn't a dodec then it is a cube.

   ```
   (frontof(a,b) --> (~dodec(a) --> cube(a))
   ```

   (c) At least one of **a** or **b** is small only if both of them are cubes.

   ```
   (small(a) v small(b)) --> (cube(a) ^ cube (b))

   Consider:  You will graduate with a CS degree only if you take CS1502.

   Some people take CS1502 and do not graduate with a CS degree, so
   graduating with a CS degree is the smaller circle, and the --> goes in
   the following direction:

   graduate with CS degree --> took CS1502.
   ```

2. (5 points) Convert the following sentence into conjunctive normal form. **Please show your work.**

$$(P \vee \neg Q) \rightarrow \neg R$$

```
(P v ~Q) --> ~R
~(P v ~Q) v ~R
(~P ^ Q) v ~R
(~P v ~R) ^ (Q v ~R)
```

3. (9 points) Give a **resolution** proof of the following argument. Please **show your work.**

1. W ∨ P ∨ Q
2. ¬Q ∨ R ∨ ¬P

—

3. W ∨ P ∨ R ∨ ¬P

```
Negate the goal:
~(W v P v R v ~P) equiv ~W ^ ~P ^ ~R ^ P

So, the clauses are:
{W,P,Q}, {~Q,R,~P}, {~W}, {~P}, {~R}, {P}

{W,P,Q} {~W}
------------
  {P,Q}  {~P}
  -----------
      {Q}  {~Q,R,~P}
      --------------
        {R, ~P}  {~R}
        ---------------
            {~P}  {P}
           -----------
             {}

The above is a good answer.  But, since it happens that the conclusion
is a tautology, there is a simpler way to answer this:

It just takes one step:  {~P}  {P}
                         ---------
```

```
                    {}
```

Why does this make sense?

Q follows from P if there is no situation in which P is true and Q is false. If Q cannot be false, it follows from anything.

I also accepted the answer

```
{W,P,Q}  {~Q,R,~P}
------------------
  {W,P,R,~P}
```

I didn't specify that you needed to do proof by refutation, so this is acceptable.

4. (10 points) For each of the following, state whether the argument is valid or invalid. If your answer is **invalid**, **show that the argument is invalid**. If your answer is **valid**, **there is nothing else to do.**

Small(b) → Rightof(c,b)
—
(Tet(b) ∨ Small(b)) → Rightof(c,b)

```
invalid
counter-example:
Small(b) is F
Rightof(c,b) is F
Tet(b) is T

premise:     F --> F, so T
conclusion:  T --> F, so F
```

Samerow(a,b) → Small(a)
Small(a)
—
Samerow(a,b)

```
invalid
counter example:
Samerow(a,b) is F
Small(a) is T

premise: F --> T, so T
conclusion: F
```

Cube(a) → Medium(a)
—
(Tet(b) ∧ Cube(a)) → Medium(a)

```
Valid
Some people gave the following as a counter example:
   a: medium cube; b a cube
But that is not a counter example.
Premise: T --> T, which is T
Conclusion: (F ^ T) --> T, which is F --> T, which is T
```

$\neg$Medium(b)
Medium(b) $\rightarrow$ Medium(c)
—
$\neg$Medium(c)

```
invalid
Counter-example:
Medium(b) is F
Medium(c) is T

premises:   ~F, so T
             F --> T, so T
conclusion: ~T, so F
```

5. Give a formal fitch-style proof of each of the following. Do **not** use any of the Con rules (Taut Con, FO Con, or Ana Con). **Include justifications, including rules and step numbers.**

   (a) (8 points)

   $(P \wedge Q) \vee R$

   —

   $R \vee P$

```
 1. (P ^ Q) v R
  ---
    |2. P ^ Q
    ----
    |3. P       and-elim 2
    |4. R v P   or-intro 3

    |5. R
    ---
    |6. R v P  or-intro 5
 7. R v P      or-elim 1,2-4,5-6
```

(b) (8 points)

$P \vee Q$
$\neg P$
—
Q

```
 1. P v Q
 2. ~P
    ----
     3.  P
         ----
     4.  _|_     _|_ intro 2,3
     5.  Q       _|_ elim 4

     6.  Q
         ---
     7.  Q        reit 6

   9. Q   v elim 1, 3-5, 6-7
```

(c) (8 points)

Tet(a) → Medium(b)
¬Medium(b)
—
¬Tet(a)

```
1.  Tet(a) --> Medium(b)
2.  ~Medium(b)
----
      3. Tet(a)
      ----
      4. Medium(b)
      5. _|_   _|_ intro 2,4
6. ~Tet(a)  neg-intro 3-5
```

(d) (10 points)

$B \to (C \to G)$
$\neg A \lor B$
$\neg A \to G$
—
$C \to G$

```
 1. B --> (C --> G)
 2. ~A v B
 3. ~A --> G
 -------------
     4.  C
     -----
         5. ~A
         -----
         6. G     -->elim 3,5

         7. B
         ----
         8. C --> G   -->elim 1,7
         9. G         -->elim 4,8
     10. G         or-elim 2,5-6,7-9
 11. C--> G    --> intro 4-10
```

6. (14 points)

Fill in the three missing steps (lines 8, 9, and 19) **and** all of the justifications in the proof on the following page. Be sure to include both the rules and step numbers. Do **not** use any of the Con rules. **Write your answers on the following page (not here.)**

See the *ppt file

7. (16 points) In each of parts (a) and (b) (on the next two pages), a truth table is given for two logical sentences.

   Determine whether the sentences are **tautologically equivalent**, whether they are **logically equivalent**, whether the second is a **tautological consequence** of the first, and whether the second is a **logical consequence** of the first.

   Support your answers by referring to the truth table. Your explanation must be specific, showing you know what the terminology means.

(a) **First sentence:** $a = b \wedge Tet(a)$ **Second sentence:** $a = b \wedge Tet(b)$

Tautologically equivalent? Circle one: **YES NO**. **Explain your answer here:**

No – in rows 2 and 3, the two sentences have different truth values.

Logically equivalent? Circle one: **YES NO**. **Explain your answer here:**

Yes. Rows 2 and 3 are spurious. In all non-spurious rows, the sentences have the same truth values.

The second is a tautological consequence of the first? Circle one: **YES NO**. **Explain your answer here:**

No. In row 2, the first is true but the second is false.

The second is a logical consequence of the first? Circle one: **YES NO**. **Explain your answer here:**

Yes. In all non-spurious rows, the second is true whenever the first is (row 1).

(b) **First sentence:** $\neg(Cube(a) \land Tet(b)) \land Cube(a)$ **Second sentence:** $Small(a) \lor \neg Tet(b)$

Tautologically equivalent? Circle one: **YES NO**. **Explain your answer here:**

No. There are rows with different truth values, for example row 1.

Logically equivalent? Circle one: **YES NO**. **Explain your answer here:**

No. There are no non-spurious rows, so no for the same reason as above.

The second is a tautological consequence of the first? Circle one: **YES NO**. **Explain your answer here:**

Yes. In both rows in which the first is true (2 and 4) the second is true as well.

The second is a logical consequence of the first? Circle one: **YES NO**. **Explain your answer here:**

Yes. In both rows in which the first is true, the second is trueas well.