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for CS2710 Foundations of AI
November 8, 2005

1 On additions and multiplications counting

Many people had questions about how to compute the number of products and additions when given an interleaved formula for computing joint probabilities. I will try to formulate the algorithm on the "syntactic parse" of the formula.

Let $A(TERM)$ denote the number of additions needed to compute the value of $TERM$. The recursive formula for computing the number of additions is:

$$A\left(\sum_{i\in I} T_i\right) = |I| * A(T_i) + |I| - 1$$

Let us analyze this formula. To get the total number of additions, we obviously need to count the additions needed in the subterms T_i. The subterms are homogeneous, so we just multiply the cost of one with the number of values the sum variable i can take on. After we have computed the values of subterms, we need to add them up, which contributes $|I| - 1$ additions. (Adding up 4 numbers takes 3 applications of $+$.)

$$A(T_1 * \ldots * T_n) = \sum_{k=1}^{n} A(T_k)$$

If our formula is a product, there are no extra additions after we compute the subterms, so we just ring up the cost of computing the subterms.

$$A(P(\ldots)) = 0$$

This is the basis of our induction, $P()$ is a table lookup (in the discrete variable case) or a density function evaluation (in the continuous case).

You can derive a similar formula for multiplication:

$$M\left(\sum_{i\in I} T_i\right) = \sum_{i\in I} M(T_i)$$

$$M(T_1 * \ldots * T_n) = n - 1 + \sum_{k=1}^{n} M(T_k)$$

$$M(P(\ldots)) = 0$$

Exercise. Derive the recursive formulas $L(TERM)$ for the number of probability table lookups.

$$L\left(\sum_{i\in I} T_i\right) = \ldots$$

$$L(T_1 * \ldots * T_n) = \ldots$$

$$L(P(\ldots)) = \ldots$$