

Problems from Chapter 3.4,3.5 and 3.6 (6th Edition) Total Points: 95**Chapter 3.4**

4. Suppose $a \mid b$, so that $b=at$ for some t , and $b \mid c$ so that $c = bs$ for some s . Then substituting the first equation into the second, we obtain $c=(at)s=a(ts)$. This means that $a \mid c$ as desired.
6. Under the hypotheses, we have $c=as$ and $d = bt$ for some s and t . Multiplying, we obtain $cd = ab(st)$, which means that $ab \mid cd$ as desired.
- 16.
- $-17 \bmod 2 = 1$
 - $144 \bmod 7 = 4$
 - $-101 \bmod 13 = 3$
 - $199 \bmod 19 = 9$
22. From $a \equiv b \pmod{m}$ we know that $b=a+sm$ for some integer s . Multiplying by c we have $bc=ac+s(mc)$, which means that $ac \equiv bc \pmod{mc}$
28. We calculate using the formula and obtain $x_1 = 13 \bmod 7 = 6$, $x_2 = 25 \bmod 7 = 4$, $x_3 = 17 \bmod 7 = 3$; and hence the sequence repeats 3,6,4,3,6,4
31. a) The message in numbers is 3-14 13-14-19 15-0-18-18 6-14. Adding 3 to each number, we obtain the numerical message 6-17 16-17-22 18-3-21-21 9-17. Then translating back we get "GR QRW SDVV JR".
- b) Similar to the above procedure, but we add 13 rather than 3 and subtract 26 from some of the terms to reduce it to modulo 26. The encoded numerical message is 16-1 0-1-6 2-13-5-5 19-1. The literal message is "QB ABG CNFF TB".
- c) This time we need to multiply 3, add 7 and reduce modulo 26 to obtain numerical message: 16-23 20-23-12 0-7-9-9 25-23. The literal message: "QX UXM AHJJ ZX".

Chapter 3.5

2. The numbers 19, 101, 107, 113 are prime, as we can verify using trial division. 27 and $93=31 \cdot 3$ are not prime.
4. We obtain the answers by trial division. The factorizations are $39=3 \cdot 13$, $81 = 3^4$, 101 is prime, $143 = 11 \cdot 13$, $289 = 17^2$, $899 = 29 \cdot 31$
- 12
- Using prime factorization: $21 = 3 \cdot 7$, $34 = 2 \cdot 17$, $55=5 \cdot 11$. Thus 21, 34, 55 are pairwise relatively prime.
 - Not relatively prime since $85 = 5 \cdot 17$
 - These are pairwise relatively prime.
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20. GCD: Find the minimum exponent for each prime factor

- a) $2^2 * 3^3 * 5^2$
- b) $2 * 3 * 11$
- c) 17
- d) 1
- e) 5
- f) $2 * 3 * 5 * 7$

22. LCM: Similar to GCD but find the maximum exponent for each prime factor.

- a) $2^5 * 3^3 * 5^5$
- b) $2^{11} * 3^9 * 5 * 7 * 11 * 13 * 17^{14}$
- c) 17^{17}
- d) $2^2 * 3^0 * 5^3 * 7 * 13$
- e) Undefined
- f) $2 * 3 * 5 * 7$

Chapter 3.5

24.

- a) $\gcd(1,5) = \gcd(1,0)=1$
- b) $\gcd(100,101) = \gcd(100,1) = \gcd(1,0) = 1$
- c) $\gcd(123,277) = 1$
- d) $\gcd(1529, 14039) = 139$
- e) $\gcd(1529, 14038) = 1$
- f) $\gcd(11111,111111)=1$

26. Divide successively by 55, 34, 21, 13, 8, 5, 3, 2, and 1. So, nine divisions are required.