

Problem 1

2.

- 1). $2^{8-1} = 128$
- 2). 7
- 3). $1 + (-1)^8 = 2$
- 4). $-(-2)^8 = -256$

4.

- 1). $a_1 = 1, a_2 = -2, a_3 = 4, a_4 = -8.$
- 2). 3, 3, 3, 3
- 3). 8, 11, 23, 71
- 4). 2, 0, 8, 0

6.

- a). 10, 7, 4, 1, -2, -5, -8, -11, -14, -17
- b). 1, 3, 6, 10, 15, 21, 28, 36, 45, 55
- c). 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025
- d). 1, 1, 1, 2, 2, 2, 2, 3, 3 (there will be $2k + 1$ copies of k).
- e). 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
- f). The largest number whose binary expansion has n bits is $(11 \dots 1)_2$, which is $2^n - 1$. So the sequence is 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023
- g). 1, 2, 2, 4, 8, 11, 33, 37, 148, 153
- h). 1, 2, 2, 2, 2, 3, 3, 3, 3, 3

8.

- (1). One rule could be that each term is 2 greater than the previous term; the sequence would be 3, 5, 7, 9, 11, . . .
- (2). Another rule is that the n^{th} term is the n^{th} prime number; the sequence would be 3, 5, 7, 11, 13, . . .
- (3). For a third rule, we could choose any number we want for the fourth term and find a third degree polynomial whose value at n would be the n^{th} term; in this case we need to solve for A,B,C, and D in $y = Ax^3 + Bx^2 + Cx + D$. When the fourth term is chosen to be 12, the formula is $(x^3 - 6x^2 + 15x - 4)/2$. With this formula, the sequence is 3, 5, 7, 12, 23, 43, 75, . . .

10.

- a). $a_0 = 3; a_n = a_{n-1} + (2n - 1)$ For all $n > 0$. The n^{th} term is $n^2 + 2$. Next 3 terms are 123, 146, 171

- b). $a_0 = 7; a_n = 7 + 4(n - 1) = 4n + 3$ For all $n > 0$. 47, 51, 55
- c). The n^{th} is the binary expansion of n . 1100, 1101, 1110
- d). One 1, three 2s, five 3s, seven 5s etc.. the value sequence is $a_0 = 1$ $a_1 = 2$
 $a_n = a_{n-1} + a_{n-2}$ For the n^{th} number, there are $2n+1$ copies appearing consecutively. 8, 8, 8
- e). The n^{th} term is $3^n - 1$. 59048, 177146, 531440
- f). We noticed that each term evenly divides the next, and the multipliers are successively 3, 5, 7, 9, 11, and so on. That must be the intended pattern
654729075, 13749310575, 316234143225
- g). The sequence consists of one 1, followed by two 0s, then three 1s, and so on, alternating between 0s and 1s and having one more item in each group than in the previous group. 0, 0, 0 (0, 0, 0)
- h). each term is the square of its predecessor.
18446744073709551616, 340282366920938463463374607431768211456,
115792089237316195435709850086879078532699846656405640394575840079
13129639936

13.

- a). $2 + 3 + 4 + 5 + 6 = 20$.
- b). $1 - 2 + 4 - 8 + 16 = 11$
- c). $3 + 3 + \dots + 3 = 10 \cdot 3 = 30$.
- d). This series "telescopes": each term cancels part of the term before it. The sum is $(2 - 1) + (4 - 2) + (8 - 4) + \dots + (512 - 256) = -1 + 512 = 511$.

18.

- a) $(1-1) + (1 - 2) + (2 - 1) + (2 - 2) + (3 - 1) + (3 - 2) = 3$
- b) $(0 + 0) + (0 + 2) + (0 + 4) + (3 + 0) + (3 + 2) + (3 + 4) + (6 + 0) + (6 + 2) + (6 + 4) + (9 + 0) + (9 + 2) + (9 + 4) = 78$
- c) $(0 + 1 + 2) + (0 + 1 + 2) + (0 + 1 + 2) = 9$
- d) $(0 + 0 + 0 + 0) + (0 + 1 + 8 + 27) + (0 + 4 + 32 + 108) = 180$

32.

- a) this set is countable. The integers in the set are 11, 12, 13, 14 ... the mapping is $1 \Leftrightarrow 11, 2 \Leftrightarrow 12, 3 \Leftrightarrow 13..$ so $n \Leftrightarrow 10 + n$
- b) this set is countable. The integers in the set are -1, -3, -5, -7 ... the mapping is $1 \Leftrightarrow -1, 2 \Leftrightarrow -3, 3 \Leftrightarrow -5..$ so $n \Leftrightarrow -(2n - 1)$
- c) uncountable. This can be proven using the same diagonalization argument as in 2.4 Example 21
- d) this set is countable. The integers in the set are 0, $\pm 10, \pm 20, \pm 30...$ the mapping is $1 \Leftrightarrow 0, 2 \Leftrightarrow 10, 3 \Leftrightarrow -10, 4 \Leftrightarrow 20...$