CS441 – Discrete Structures for Computer Science
Instructor: Milos Hauskrecht

Problems from Section 2.1, 2.2 and 2.3
Total Points: 100

Section 2.1

4. **(4 points)**

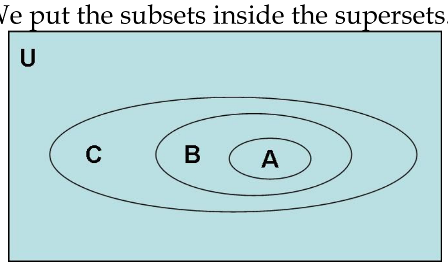
Each of the set is a subset of itself. Also, B is a subset of A and C is a subset of A and D.

9. **(6 points)**

a). T (in fact x is the only element) b). T (every set is a subset of itself)
c). F (the only element of {x} us a letter, not a set)
d). T (in fact, {x} is the only element)
e). T (the empty set is a subset of every set)
f). F (the only element of {x} is a letter, not a set)

12. **(5 points)**

We put the subsets inside the supersets. Thus the answer is as shown:



17. **(4 points)**

a) 1
b) 1
c) 2
d) 3 (Elements of the set: a, {a} and {a, {a}})

28. In each case, the answer is a set of 3-tuples. **(8 points)**

a). {(a,x,0), (a,x,1), (a,y,0), (a,y,1), (b,x,0), (b,x,1), (b,y,0), (b,y,1), (c,x,0), (c,x,1), (c,y,0), (c,y,1)}
b). {(0,x,a), (0,x,b), (0,x,c), (0,y,a), (0,y,b), (0,y,c), (1,x,a), (1,x,b), (1,x,c), (1,y,a), (1,y,b), (1,y,c)}
c). {(0,a,x), (0,a,y), (0,b,x), (0,b,y), (0,c,x), (0,c,y), (1,a,x), (1,a,y), (1,b,x), (1,b,y), (1,c,x), (1,c,y)}
d). {(x,x,x), (x,x,y), (x,y,x), (x,y,y), (y,x,x), (y,x,y), (y,y,x), (y,y,y)}

Section 2.2

4. Note that A is a subset of B. **(8 points)**

a) {a,b,c,d,e,f,g,h}=B
b) {a,b,c,d,e}=A
c) ϕ (There are no elements in A that are not in B)
d) {f,g,h}

12. **(4 points)**

We will show that these two sets are equal by showing that each is a subset of the other. Suppose $x\in A\cup(A\cap B)$. Then $x\in A$ or $x\in A\cap B$ by definition of union. In the former case we have $x\in A$, and in the latter case we have $x\in A$ and $x\in B$ by definition of intersection; thus in any case $x\in A$. Hence we have proved that LHS (left-hand side) is a subset of RHS. Conversely let $x\in A$. Then by definition of union, $x \in A \cup (A \cap B)$ as well. Thus we have shown that have proved that RHS (right-hand side) is a subset of LHS.

14. **(6 points)**

$A=(A-B) \cup (A \cap B)$, $A = \{1,3,5,6,7,8,9\}$. Similarly, $B=(B-A) \cup (A \cap B)$, $B = \{2,3,6,9,10\}$.

16. **(10 points)**

a) If x is in $A \bigcap B$, then perforce it is in A (by definition of intersection)
b) If x is in A, then perforce it is in $A \bigcup B$ (by definition of union)
c) If x is in $A - B$, then perforce it is in A (by definition of difference)
d) If $x \in A$ then $x \notin B - A$. Therefore there can be no elements in $A \bigcap (B - A)$, so $A \bigcap (B - A) = \phi$
e) The left-hand side consists precisely of those things that are either elements of *A* or else elements of *B* but not *A*, in other words, things that are elements of either *A* or *B* (or, of course, both). This is precisely the definition of the right-hand side

25. **(8 points)**

a) {4,6}
b) {0,1,2,3,4,5,6,7,8,9,10}
c) {4,5,6,8,10}
d) {0,2,4,5,6,7,8,9,10}

50. **(6 points)**

a) 00 1110 0000 b). 10 1001 0001 c). 01 1100 1110

Section 2.3

2. **(6 points)**

a) No (Rule is not very well denied. We do not know whether f(3)=3 or f(3)=-3).
b)Yes (For all integers n, $\sqrt{(n^2+1)})$ is a well-defined real number.)
c) No (f(n) is undefined for n=2 and n=-2)

6. **(10 points)**

a) Domain: $Z^+ \times Z^+$ and Range: Z^+
b) Since the largest decimal digit of a strictly positive integer cannot be 0, Domain: Z^+ and Range:{1,2,3,4,5,6,7,8,9}
c) The domain is the set of all bit strings. The number of 1's minus number of 0's can be any positive or negative or 0, so range is Z.
d) Domain: Z^+ and Range: Z^+.
e) The domain is the set of all bit strings. The range is set of strings of 1's:{ λ,1,11,111,…}, where λ is the empty string.

10. **(6 points)**

a) Yes
b) No (Since b is the image of both a and b.)
c) No (Since d is the image of both a and d.)

14. **(5 points)**

a) Yes (f(0,-n)= n for every integer n)
b) No (Since 2 is not in the range. To see this, $m^2-n^2= (m - n)(m + n)$ can only be even if m and n are both even or both odd. In both case (m - n) and (m + n) are even integers and their product is divisible by 4 and cannot equal 2.
c) Yes (Since f(0,n-1)=n for every integer n.
d) Yes (To achieve negative values we set m=0 and for nonnegative values we set n=0)
e) No

18. **(4 points)**

a) Yes (Since inverse function is (4-x)/3)

b) No (Not one-to-one as f(17)=f(-17), for instance and also not onto.)

c) No (This is bijection, but not from R to R. And x=-2 is not in the domain and x=1 is not in range (Inverse not defined). It is a bijection from R-{-2} to R-{1} and its inverse clearly is (1-2x)/(x-1))

d) Yes (It is clear that this function is increasing throughout its domain (R) and it takes on both arbitrarily large values and arbitrarily small values (large negative) values. So it is a bijection. Its inverse is clearly $(x-1)^{1/5}$.)