

## Problems from Section 1.4, 1.5, 1.6 and 1.7 (6th Edition)

Total Points: 80

## Section 1.4

4. (12 points)

- Some student in your class has taken some computer science course.
- There is a student in your class who has taken every computer science course.
- Every student in your class has taken at least one computer science course.
- There is a computer science course that every student in your class has taken
- Every computer science course has been taken by at least one student in your class.
- Every student in your class has taken every computer science course.

8. (10 points)

- $\exists x \exists y Q(x, y)$
- $\forall x \forall y \neg Q(x, y)$
- $\exists x (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel Of Fortune}))$
- $\forall y \exists x Q(x, y)$
- $\exists x \exists y (Q(x, \text{Jeopardy}) \wedge Q(y, \text{Jeopardy}) \wedge x \neq y)$

10. (12 points)

- $\forall x F(x, \text{Fred})$
- $\forall y F(\text{Evelyn}, y)$
- $\forall x \exists y F(x, y)$
- $\neg \exists x \forall y F(x, y)$
- $\forall y \exists x F(x, y)$
- $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

## Section 1.5

4. (5 points)

- Simplification
- Disjunctive syllogism
- Modus ponens
- Addition
- Hypothetical syllogism

6. (5 points)

- r: it rains  
 f: it is foggy  
 s: the sailing race will be held  
 l: the life saving demonstration will go on  
 t: the trophy will be awarded
- | Step                 | Reason     |
|----------------------|------------|
| 1. $\neg t$          | Hypothesis |
| 2. $s \rightarrow t$ | Hypothesis |

- |  |                             |
|--|-----------------------------|
| 3. $\neg s$  | Modus tollens using 1 and 2 |
| 4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$           | Hypothesis                  |
| 5. $(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$ | Contrapositive of step 4    |
| 6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$           | De Morgan's law             |
| 7. $(\neg s \vee \neg l)$                                    | Addition using 3            |
| 8. $r \wedge f$  | Modus ponens using 6 and 7  |
| 9. $r$   | Simplification using 8.     |

10. (12 points)
- a) If we use modus tollens starting from back, then we conclude that "I am not sore". Another application of modus tollens then tells us that "I did not play hockey".
- b) We cannot conclude anything specific here.
- c)
1. Dragon flies have six legs by universal instantiation and modus ponens of first implication.
  2. Spiders are not insects (modus tollens)
  3. There exists a non-six-legged creature that eats a six-legged creature. (Existential generalization)
  4. There exists a non-insect that eats an insect. (Existential generalization)
- d) We can apply universal instantiation to the conditional statement and conclude that if Homer (respectively, Maggie) is a student, then he (she) has an internet account. Now modus tollens tells us that "Homer is not a student". There are no conclusions to be drawn about Maggie.
- e) The first conditional statement is that if  $x$  is healthy to eat, then  $x$  does not taste good. Universal instantiation and modus ponens therefore tell us that "Tofu does not taste good". The third sentence says that if you eat  $x$ , then  $x$  tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.
- f) By disjunctive syllogism, the first two hypotheses allow us to conclude that I am hallucinating. Therefore by modus ponens we know that "I see elephants running down the road".

### Section 1.6

2. (4 points)
- To show that whenever we have two even integers, their sum is also even. Suppose that  $a$  and  $b$  are two even integers. Then there exist integers  $s$  and  $t$  such that  $a=2s$  and  $b = 2t$ . Adding, we obtain  $a+b = 2s+2t=2(s+t)$ . We conclude that  $a+b$  is even.
18. (8 points)
- a) To prove contrapositive: If  $n$  is odd, then  $3n+2$  is odd. Assume that  $n$  is odd. Then we can write  $n=2k+1$  for some integer  $k$ . Then  $3n+2 = 3(2k+1) = 2(3k+2) + 1$ . Thus  $(3n+2)$  is odd.
- b) To prove contradiction: Suppose that  $(3n+2)$  is even and that  $n$  is odd. Since  $3n+2$  is even, so is  $3n$ . If we add subtract an odd number from an even number, we get an odd number,  $3n-n=2n$  is odd. But this is not true. Therefore our supposition was wrong, and the proof by contradiction is complete.
20. (4 points)
- We need to prove the proposition "If  $1$  is a positive integer, then  $1^2 \geq 1$ ". The conclusion is the true statement  $1 \geq 1$ . Therefore the conditional statement is true. This is an example of a trivial proof.

30.

**(4 points)**

We are given (i)  $a < b$ , (ii)  $(a+b)/2 > a$ , (iii)  $(a+b)/2 < b$ . The latter two are equivalent to  $(a+b) > 2a$  and  $(a+b) < 2b$ , respectively, and these in turn equivalent to  $b > a$  and  $a > b$ , respectively. Hence all three statements are equivalent.

### **Section 1.7**

6.

**(4 points)**

The number 1 has this property, since the only positive integer not exceeding 1 is 1 itself, and therefore the sum is 1. This is a constructive proof. It is important to note the number itself is included in the sum if not exceeding itself.