**CS441 – Discrete Structures for Computer Science**

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**Section 1.2 solutions**

**6.**

| p | q | $p \wedge q$ | $\neg(p \wedge q)$ | $\neg q \vee \neg p$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

8.

a.) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Kwame will not take a job in industry and (Kwame) will not go to graduate school.
Kwame will neither take a job in industry nor will he go to graduate school.

Common error 1) *Kwame will not take a job in industry and go to grad school.*

This (ambiguous) statement is could be read as $\neg(p \wedge q)$ or $\neg p \wedge q$. In particular, $\neg(p \wedge q)$, is still true if either p or q happens, also long as both do not, however the negation of the original statement should imply neither will occur.

b) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

Yoshiko does not know Java or (Yoshiko) does not know Calculus.

Common error 1) *Yoshiko does not know Java or Calculus.*

This statement reads $\neg(p \vee q)$, implying that neither is the case.
The moral of the English inuition (what may seem natural) is different that the formal definition for Logic (DeMorgan's).

Common error 2) *Yoshiko knows neither Java nor Calculus.*

This statement reads $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$.

c) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

James is not young or (James is) not strong.

d) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Rita will not move to Oregon and (Rita will) not move to Washington.

14.

| p | Q | $p \Rightarrow q$ | $\neg p \wedge (p \Rightarrow q)$ | $(\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg q$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | T |
| F | T | T | F | F |
| F | F | T | T | T |

This truth table shows that the proposition $(\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg q$ is not a tautology because the last column is not always true.

Using equivalence rules:

$(\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg q \equiv \neg ((\neg p \wedge (p \Rightarrow q)) \vee \neg q$ (by def. of implication)

$\equiv p \vee \neg( p \Rightarrow q) \vee \neg q$ (by DeMorgan;s law)

$\equiv p \vee \neg( \neg p \vee q) \vee \neg q$ (by def. of implication)

$\equiv p \vee (p \wedge \neg q) \vee \neg q$ (by DeMorgan;s law)

$\equiv (p \wedge \neg q) \vee p \vee \neg q$ (by commutative law)

$\equiv p \vee \neg q$ (by domination, $\vee$ over $\wedge$)

not a tautology because this proposition is false when p is false and q is true

16.

| p | Q | $p \Leftrightarrow q$ | $p \wedge q$ | $\neg p \wedge \neg q$ | $(p \wedge q) \vee (\neg p \wedge \neg q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | T |
| T | F | F | F | F | F |
| F | T | F | F | F | F |
| F | F | T | F | T | T |

Column 3 and column 6 are equal, so they are logically equivalent

$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ (by def. of the biconditional)

$\equiv ( \neg p \vee q) \wedge ( \neg q \vee p)$ (by def. of implication)

$\equiv (\neg p \wedge \neg q) \vee ( q \wedge p)$ (by distributive law)

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (by commutativity)

Common Mistake 1) assuming $p \Leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ via table 8. This is what you are being asked to demonstrate, using the decomposition into the conjunction of two implications (see hint).

20.

| P | Q | $p \Leftrightarrow q$ | $p \oplus q$ | $\neg(p \oplus q)$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | F |
| F | T | F | T | F |
| F | F | T | F | T |

Column 3 and column 5 are equal, so they are logically equivalent

Note: $p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$

The equivalence follows from the equivalence in 16 (above) by the application of DeMorgan’s law.

40.

$(p \wedge q \wedge \neg r)$

## Section 1.3.

**Problem 2 (Section 1.3)**

2.

a)True b) False c) False d) True

4.

a) 0 (condition is false)
b) 1 (condition is false)
c) 1 (condition is true $x$:=1 executed)

6.

a) Some students visited ND
b) Every student has visited ND
c) No student has visited ND
d) At least one student has not visited ND
e) Not all students visited ND

Common Error: interpreting $\neg\forall x(N(x))$, as $\forall x(\neg N(x))$. Instead, $\neg\forall x(N(x))$ could be read as $\exists x\,(\neg N(x))$, so it is incorrect to assume no one has been to ND

f) No student has visited ND

8.

a) if an animal is a rabbit, then it hops
b) Every animal is a rabbit and hops
c) There exists an animal such that if it is a rabbit, then it hops

Common error : *There exist rabbits that hop*. Remember that if the condition for the implication is false, the implication is still true., so if all animals are not rabbits, the propisition holds.

d) Some rabbits hop

10. a) $\exists x\,(C(x) \wedge D(x) \wedge F(x))$
b) $\forall x\,(C(x) \vee D(x) \vee F(x))$

Common error: using *conjunction* rather than disjunction. All students own at least one, but not necessarily all 3 kinds of pets

c) $\exists x\,(C(x) \wedge \neg D(x) \wedge F(x))$
d) $\neg\exists x\,(C(x) \wedge D(x) \wedge F(x))$

Common error: $\forall x\,(\neg C(x) \wedge \neg D(x) \wedge \neg F(x))$. The proper equivalence is

$$\forall x\,(\neg C(x) \vee \neg D(x) \vee \neg F(x))$$

e) $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$.

Common Error : Note that the owners of the pets may be different, so a *single existential* variable is insufficient.

Common Error : using *xors*, It is also possible that two or more pets are owned exclusively by a single student, the proposition still holds.

12.

a) True
b) True
c) False
d) True
e) False
f) True
g) False

16.

a) True (square root of 2)
   Common error: note the domain is reals not integers.
b) False
c) True
d) False

24. C(x): x is in your class

a) P(x): x has a cell phone
   1. $\forall x\, (P(x))$
   2. $\forall x\, (C(x) \Rightarrow P(x))$
b) F(x): x has seen a foreign movie
   1. $\exists x\, (F(x))$
   2. $\exists x (C(x) \wedge F(x))$

c) S(x): x can swim
   1. $\exists x\, (\neg S(x))$
   2. $\exists x (C(x) \wedge \neg S(x))$

d) Q(x): x can solve quadratic equations
   1. $\forall x\, (Q(x))$
   2. $\forall x\, (C(x) \Rightarrow Q(x))$
e) R(x): x wants to be rich
   1. $\exists x\, (\neg R(x))$
   2. $\exists x\, (C(x) \wedge \neg R(x))$

36 .

a) 0 and 1 are counterexamples
b) square root of 2 and negative square root of 2 are counterexamples
c) 0 is a counterexample