CS 2750 Machine Learning

Lecture 2

Designing a learning system

Milos Hauskrecht
milos@cs.pitt.edu
5329 Sennott Square, x4-8845

people.cs.pitt.edu/~milos/courses/cs2750/

Administrivia

- **No homework assignment this week**

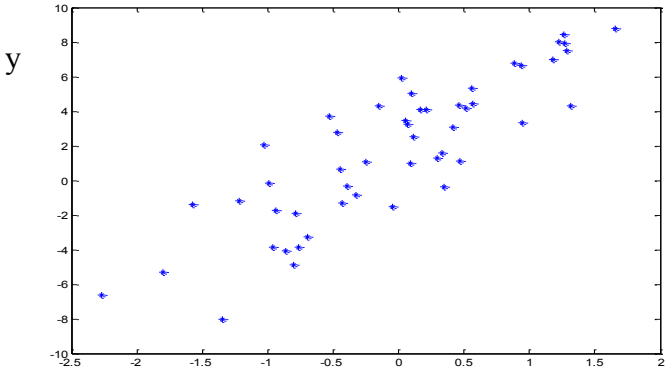
- Please try to obtain a copy of Matlab:
  http://technology.pitt.edu/software/matlab-students
- A brief tutorial on Matlab next week

Learning: first look

Assume we get a dataset D that consists of pairs (**x** , y)

Goal: learn the mapping $f : X \to Y$ to is able to predict well y for some future **x.**

Question: How do we learn f ?



Learning: first look

1. Data: $D = \{d_1, d_2, .., d_n\}$

2. Model selection:

- **Select a model** or a set of models (with parameters)

  E.g. $y = ax + b$

3. Choose the objective (error) function

- **Squared error** $Error(D, a, b) = \frac{1}{n}\sum_{i=1}^{n}(y_i - ax_i - b))^2$

4. Learning:

- **Find the set of parameters (*a,b*) optimizing the error function**

$$(a^*, b^*) = \arg\max_{(a,b)} Error(D, a, b)$$

5. Application

- **Apply the learned model to new data** $f(x) = a^* x + b^*$
- E.g. predict ys for the new input x

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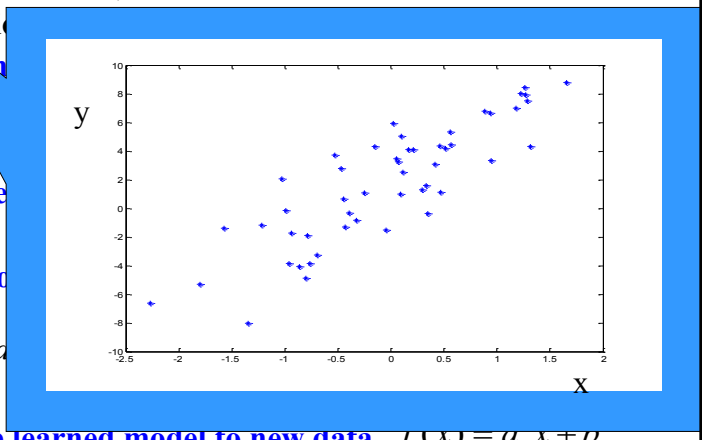

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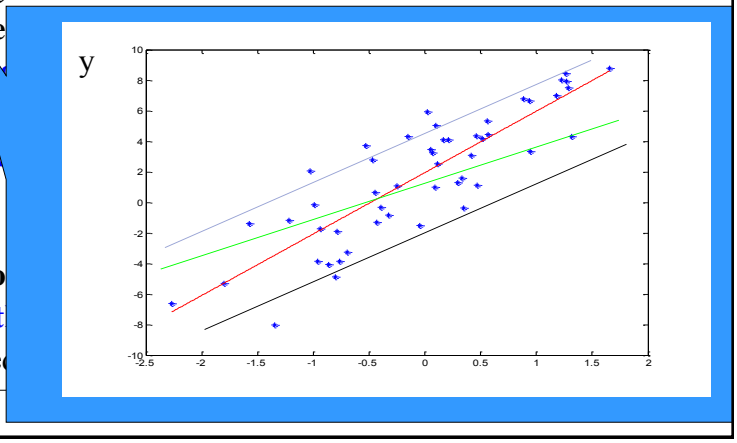

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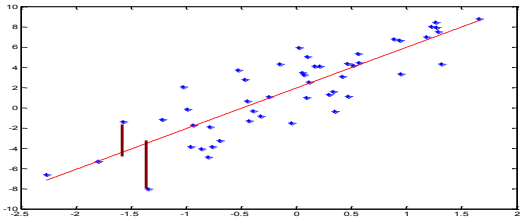

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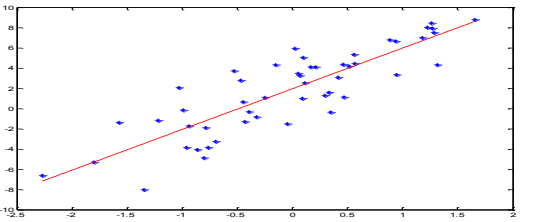

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- **Find the set of parameters (*a*,*b*) optimizing the error function**

$$(a^*, b^*) = \arg\max_{(a,b)} Error(D, a, b)$$

5. Application

- **Apply the learned model to new data** $f(x) = a^* x + b^*$
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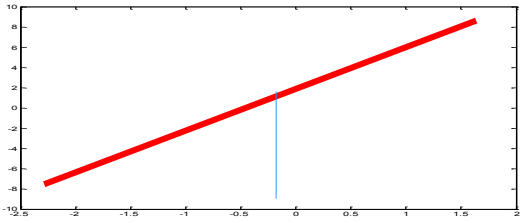

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– **Apply the learned model to new data** $f(x) = a^* x + b^*$

Looks straightforward, but there are problems ….

–

Learning: generalization error

We fit the model based on past examples observed in ***D***

Training data: **Data used to fit the parameters of the model**

Training error:

$$Error(D,a,b) = \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$$

Problem: Ultimately we are interested in learning the mapping that performs well on the whole population of examples

True (generalization) error (over the whole population):

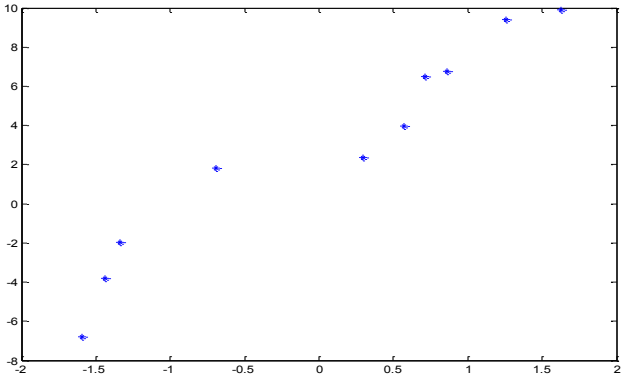
$$Error(a,b) = E_{(x,y)}[(y - f(x))^2]$$ Mean squared error

Training error tries to approximate the true error !!!!

Does a good training error imply a good generalization error ?

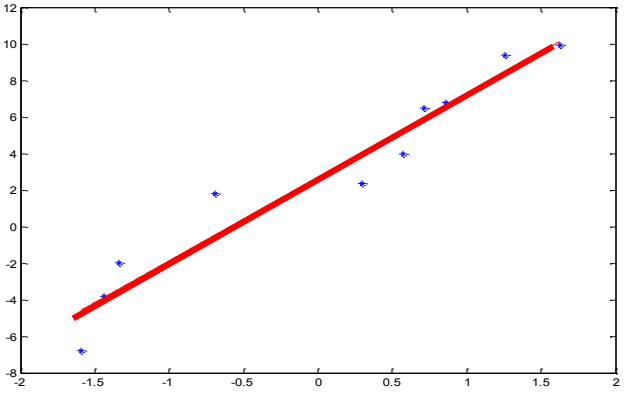
Overfitting

- Assume we have a set of 10 points and we consider polynomial functions as our possible models



Overfitting

- Fitting a linear function with the square error
- **Error is nonzero. Why?**



Overfitting

- Fitting a linear function with the square error
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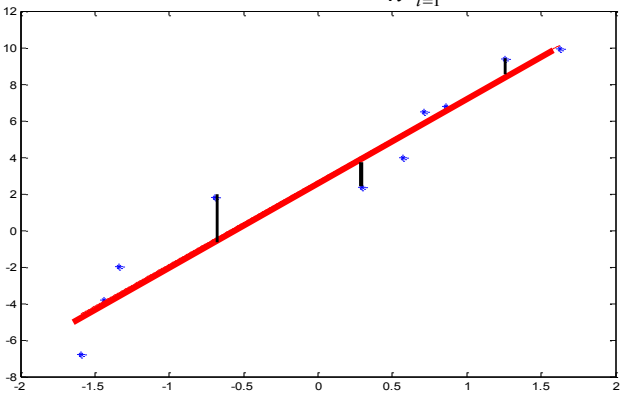
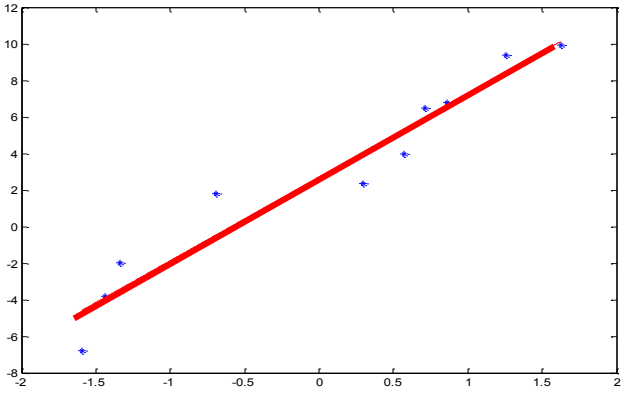


Overfitting

Assume in addition to a linear model: $y = f(x) = ax + b$

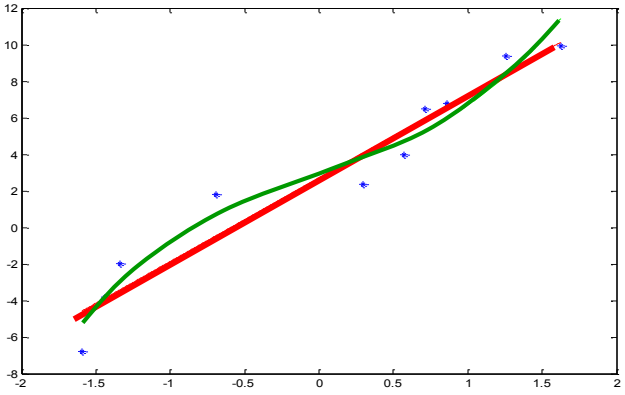
also: $y = f(x) = a_3x^3 + a_2x^2 + a_1x + b$

Which model would give us a smaller error for the least squares fit?



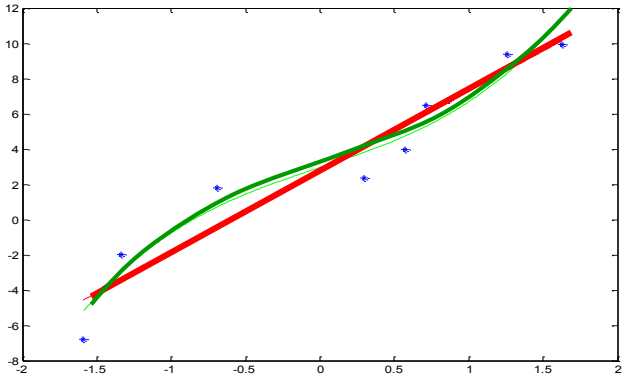
Overfitting

- Linear vs. cubic polynomial
- Higher order polynomial leads to a better fit, smaller error



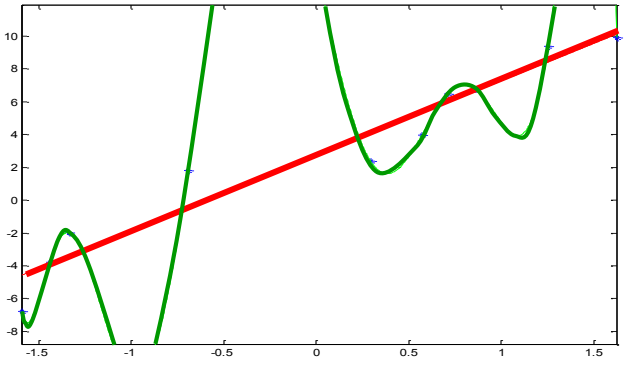
Overfitting

- Is it always good to minimize the error of the observed data?



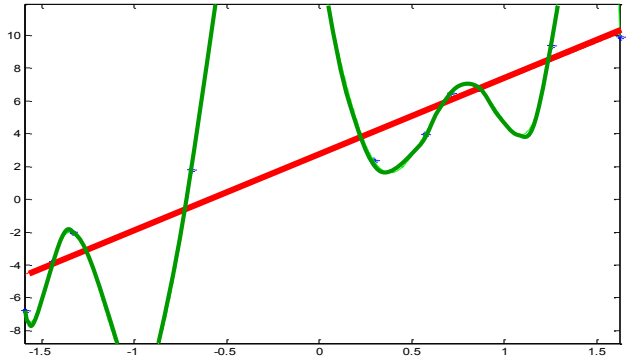
Overfitting

- For 10 data points, the degree 9 polynomial gives a perfect fit (Lagrange interpolation). Error is zero.
- Is it always good to minimize the training error?



Overfitting

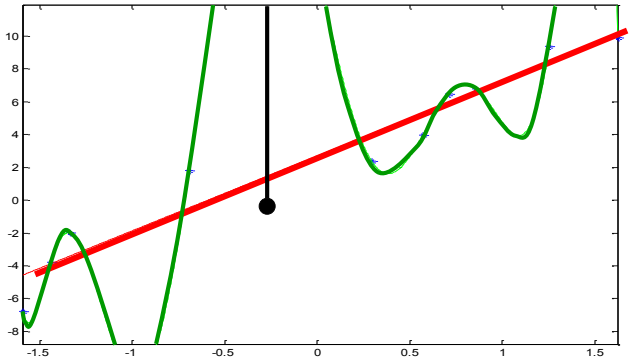
- For 10 data points, degree 9 polynomial gives a perfect fit (Lagrange interpolation). Error is zero.
- Is it always good to minimize the training error? NO !!
- **More important:** How do we perform on the unseen data?



Overfitting

Situation when the training error is low and the generalization error is high. Causes of the phenomenon:

- Model with a large number of parameters (degrees of freedom)
- Small data size (as compared to the complexity of the model)



How to evaluate the learner's performance?

- **Generalization error** is the true error for the population of examples we would like to optimize

$$E_{(x,y)}[(y - f(x))^2]$$

- But it cannot be computed exactly
- **Sample mean only approximates the true mean**

- **Optimizing the training error can lead to the overfit, i.e.** training error may not reflect properly the generalization error

$$\frac{1}{n}\sum_{i=1,..n}(y_i - f(x_i))^2$$

- So how to assess the generalization error?

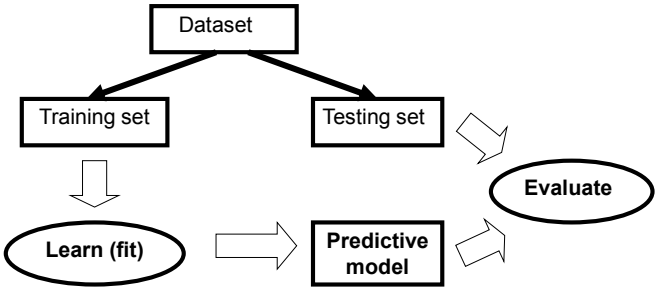
How to evaluate the learner's performance?

- **Generalization error** is the true error for the population of examples we would like to optimize
- **Sample mean only approximates it**
- **Two ways to assess the generalization error is:**
  - **Theoretical: Law of Large numbers**
    - statistical bounds on the difference between true and sample mean errors
  - **Practical:** Use a separate data set with *m* data samples to test the model
    - **(Average) test error**

$$Error(D_{test}, f) = \frac{1}{m}\sum_{j=1,..m}(y_j - f(x_j))^2$$

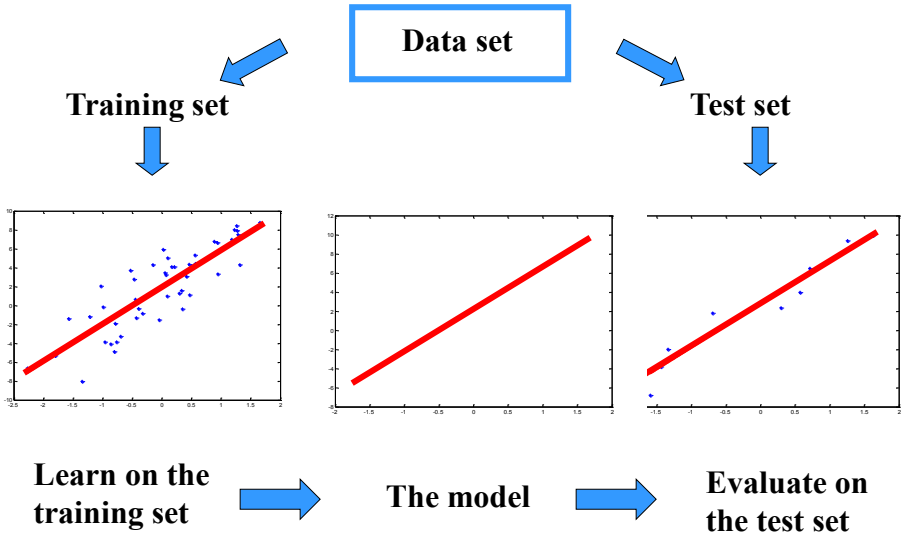
Assessment of the generalization performance

- **Simple holdout method**
  - Divide the data into the disjoint training and test data

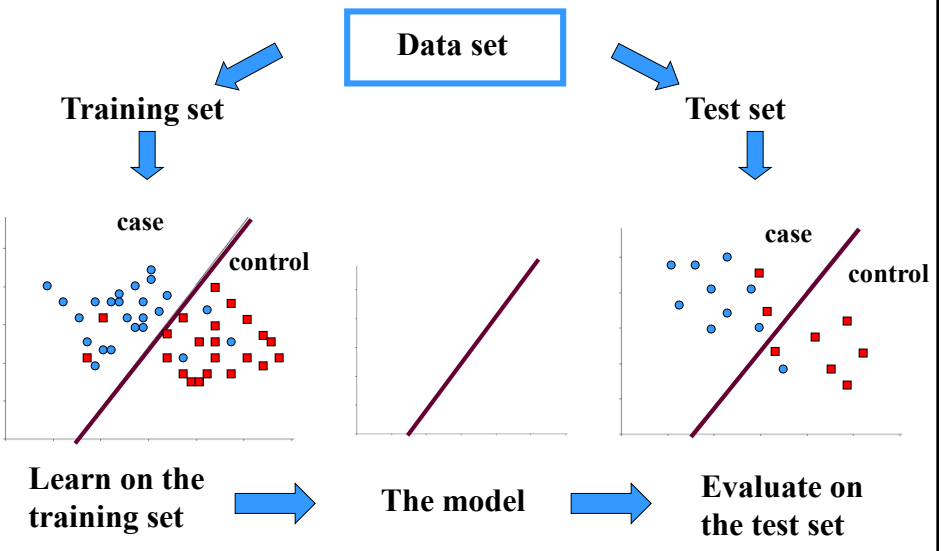


  - Typically 2/3 training and 1/3 testing

Testing of models: regression



Testing of models: classification



Evaluation measures

Easiest way to evaluate the model:

- **Error function used in the optimization is adopted also in the evaluation**
- **Advantage: may help us to see model overfitting. Simply compare the error on the training and testing data.**

Evaluation of the models often considers:

- **Other aspects or statistics of the model and its performance**
- Moreover the Error function used for the optimization may be a convenient approximation of the quality measure we would really like to optimize

Evaluation measures

Classification:

		Actual: Case	Actual: Control
Prediction	Case	TP 0.3	FP 0.1
	Control	FN 0.2	TN 0.4

Misclassification error:

$$E = FP + FN$$

Sensitivity:

$$SN = \frac{TP}{TP + FN}$$

Specificity:

$$SP = \frac{TN}{TN + FP}$$

A learning system: basic cycle

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4. Learning:

- **Find the set of parameters optimizing the error function**
  - The model and parameters with the smallest error

5. Testing/validation:

- **Evaluate on the test data**

6. Application

- **Apply the learned model to new data** $f(\mathbf{x})$

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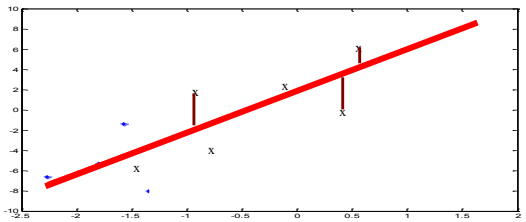

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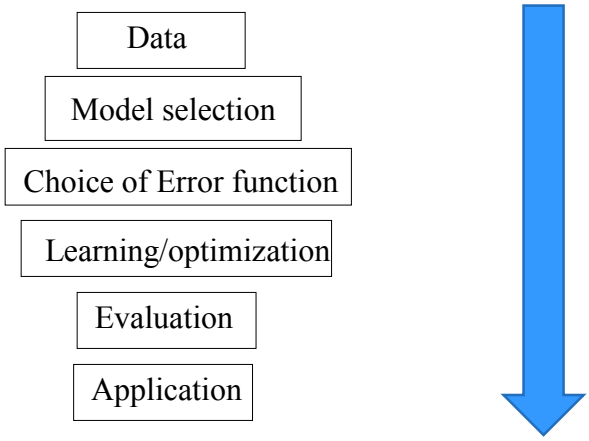
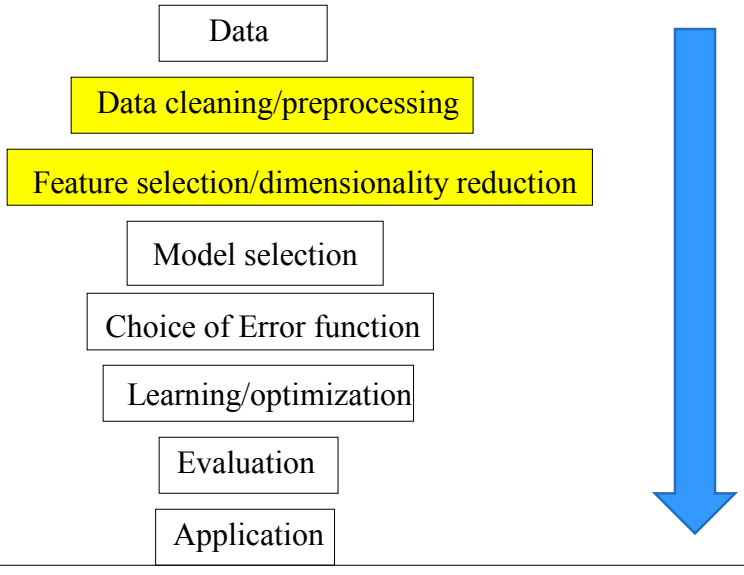
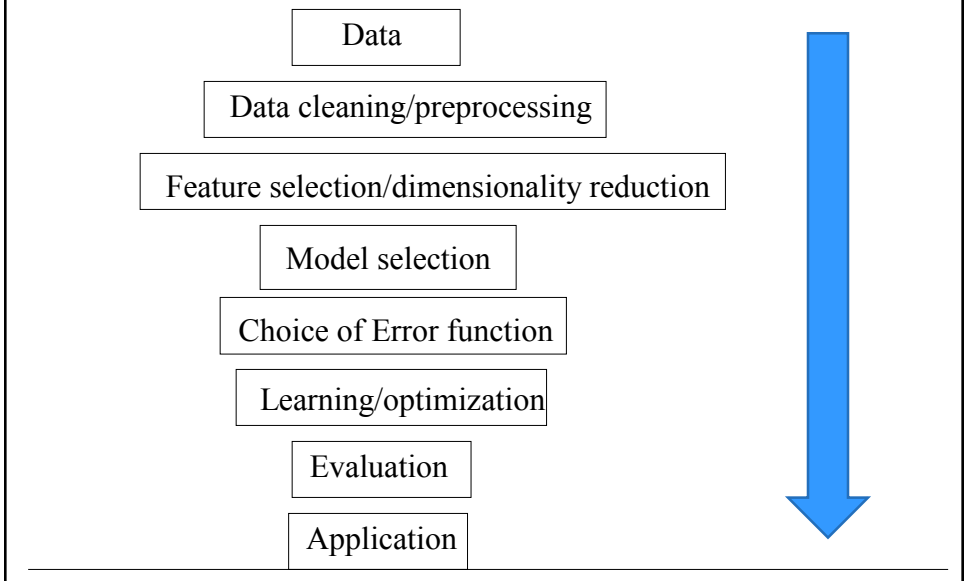
Steps taken when designing an ML system



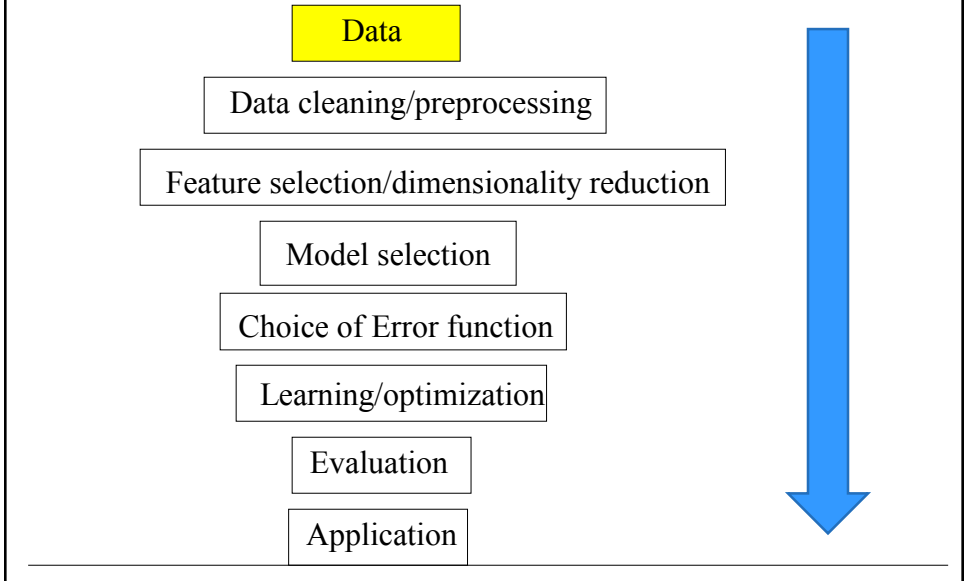
Add some complexity



Designing an ML solution



Designing an ML solution



Data source and data biases

- **Understand the data source**
- **Understand the data your models will be applied to**
- **Watch out for data biases:**
  - Make sure the data we make conclusions on are the same as data we used in the analysis
  - It is very easy to derive "unexpected" results when data used for analysis and learning are biased
- **Results (conclusions) derived for a biased dataset do not hold in general !!!**

Data biases

Example: Assume you want to build an ML program for predicting the stock behavior and for choosing your investment strategy

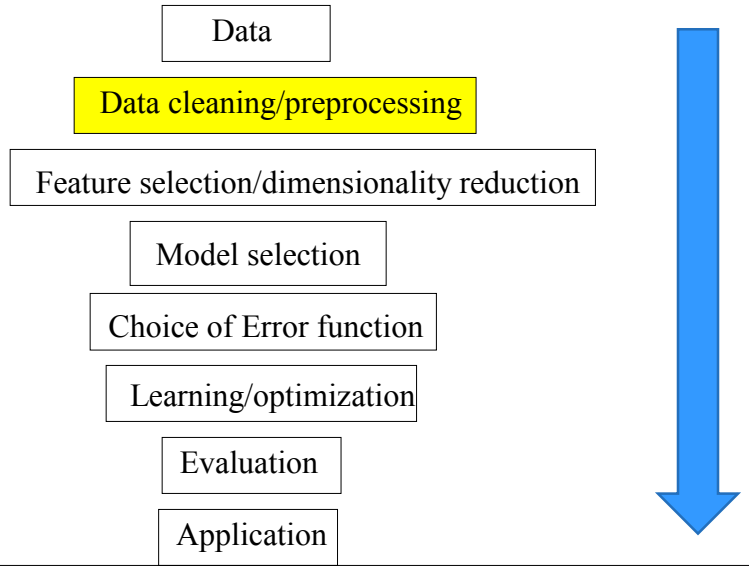
Data extraction:

- pick companies that are traded on the stock market on January 2017
- Go back 30 years and extract all the data for these companies
- Use the data to build an ML model supporting your future investments

Question:

- **Would you trust the model?**
- **Are there any biases in the data?**

Steps taken when designing an ML system



Data cleaning and preprocessing

Data you receive may not be perfect:

- **Cleaning**
- **Preprocessing (conversions)**

Cleaning:

- **Get rid of errors, noise,**
- **Removal of redundancies**

Preprocessing:

- **Renaming**
- **Rescaling (normalization)**
- **Discretizations**
- **Abstraction**
- **Aggregation**
- **New attributes**

Data preprocessing

Renaming (relabeling) categorical values to numbers

- dangerous in conjunction with some learning methods
- numbers will impose an order that is not warranted

High → 2
Normal → 1
Low → 0

True → 2
False → 1
Unknown → 0

Red → 2
Blue → 1
Green → 0

Problem: How to safely represent the different categories as numbers when no order exists?

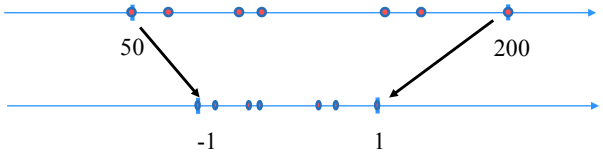
Solution: Use indicator vector (or one-hot) representation.

- **Example: Red, Blue, Green colors**

  3 categories → use a vector of binary (0,1) values of size 3
  Encoding: **Red:** (1,0,0); **Blue:** (0,1,0); and **Green:** (0,0,1)

Data preprocessing

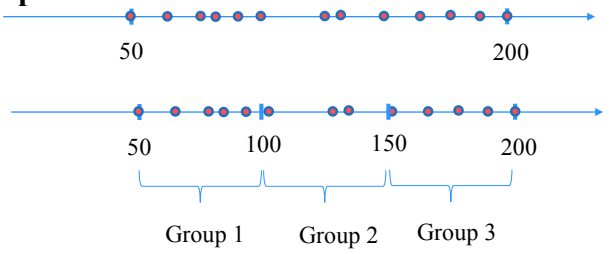
- **Rescaling (normalization):** continuous values transformed to some range, typically [-1, 1] or [0,1].



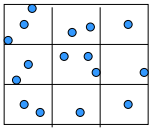
- Why normalization?
  - Some learning algorithms are sensitive to the values recorded in the specific input field and its magnitude

Data preprocessing

- **Discretization (binning):** continuous values to a finite set of discrete values
- **Example:**



- **Example 2:**



Data preprocessing

- **Abstraction:** merge together categorical values
- **Aggregation:** summary or aggregation operations, such minimum value, maximum value, average etc.
- **New attributes:**
  - example: obesity-factor = weight/height