CS 2750 Machine Learning
Lecture 12b

Decision trees

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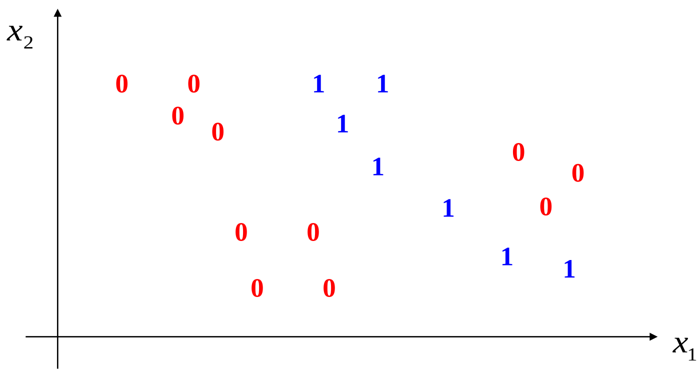
Decision tree classification

- An alternative approach to classification:
  - **Partition the input space to regions**
  - **Regress or classify independently in every region**



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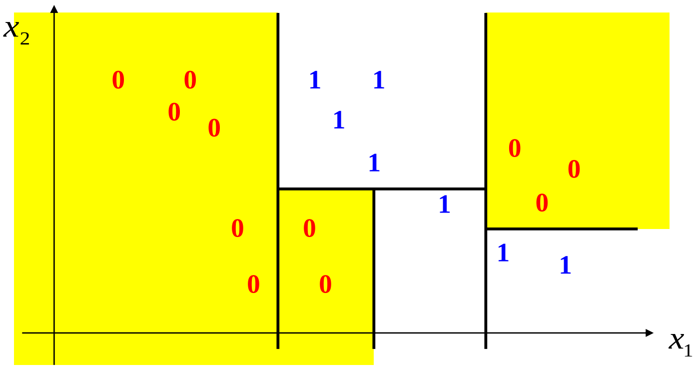


Decision tree classification

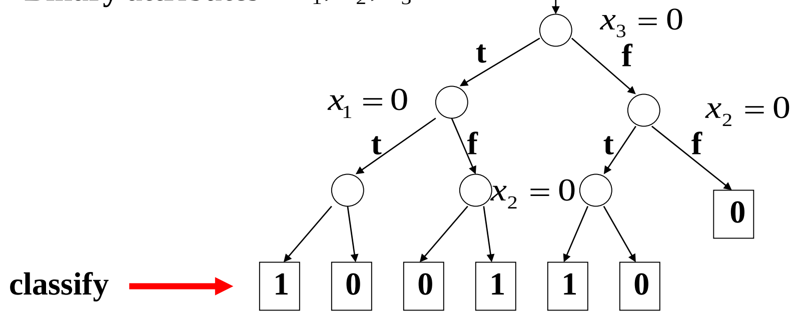
Decision tree model:

- Split recursively the input space **x** using simple conditions on x_i
- Classify at the bottom of the tree

Example:

Binary classification $\{0,1\}$

Binary attributes x_1, x_2, x_3



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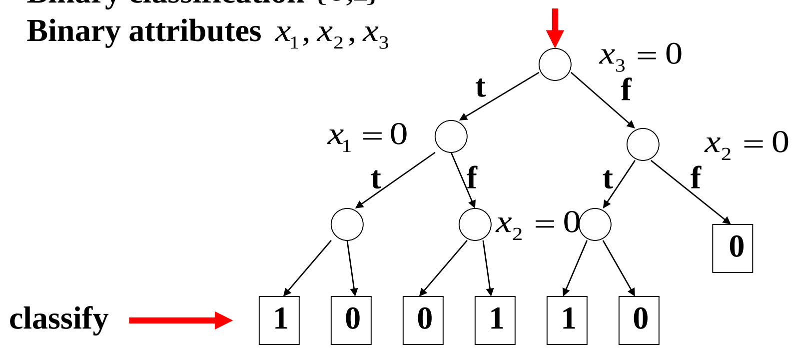

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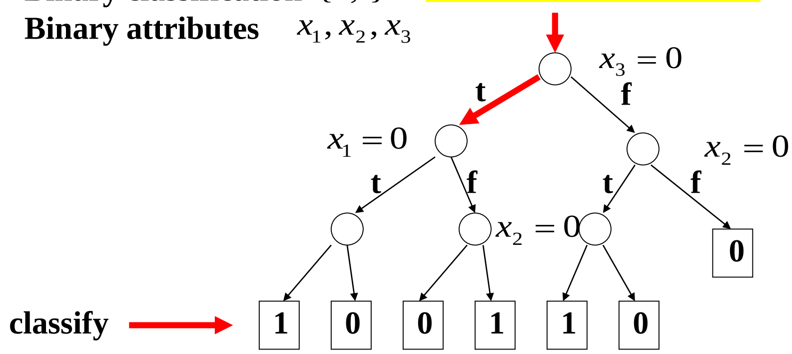

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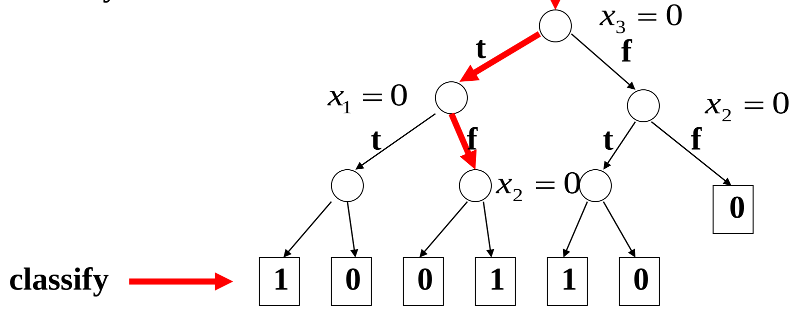

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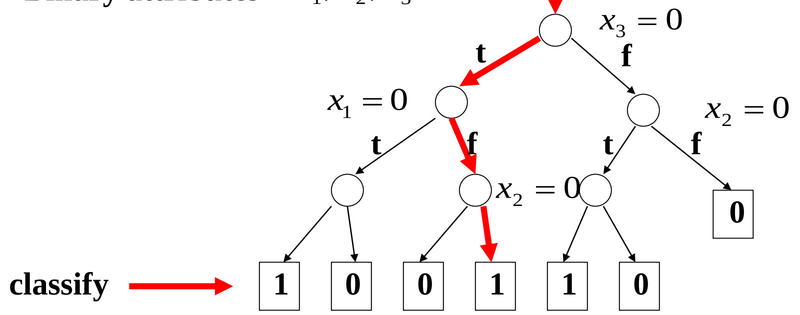
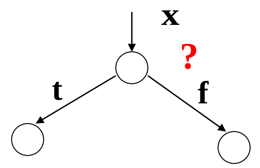


Learning decision trees

How to construct /learn the decision tree?

- **Top-bottom algorithm**:
  – Find the best split condition (quantified based on the **impurity measure**)
  – Stops when no improvement possible



- **Impurity measure *I*(D)**:
  – measures the degree of mixing of the two classes in the subset of the training data D
  – Worst (maximum impurity) when # of 0s and 1s is the same
- Splits: **finite or continuous value attributes**

Continuous value attributes conditions: $x_3 \leq 0.5$

Impurity measure

Let $|D|$ **- Total number of data instances in D**

$|D_i|$ **- Number of data entries classified as *i***

$p_i = \frac{|D_i|}{|D|}$ **- ratio of instances classified as *i***

Impurity measure *I*(D)

- Measures the degree of mixing of the two classes in D
- The impurity measure should satisfy:
  – Largest when data are split evenly for attribute values

$$p_i = \frac{1}{\text{number of classes}}$$

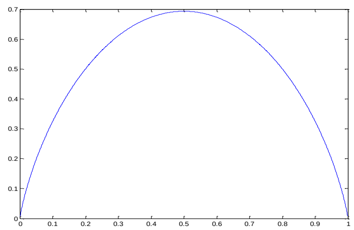
  – Should be 0 when all data belong to the same class

Impurity measures

- There are various impurity measures used in the literature
  - **Entropy based measure (Quinlan, C4.5)**

$$I(D) = Entropy(D) = -\sum_{i=1}^{k} p_i \log p_i$$

Example for k=2



  - **Gini measure (Breiman, CART)**

$$I(D) = Gini(D) = 1 - \sum_{i=1}^{k} p_i^{\,2}$$

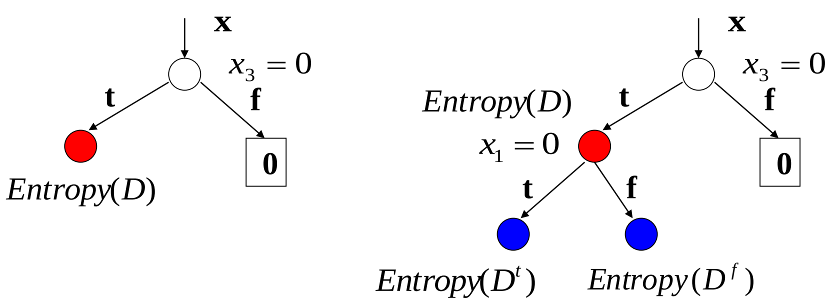
Impurity measures

- **Gini due to split** – expected reduction in the impurity measure (entropy example)

Split condition

$$Gain(D, A) = Entropy(D) - \sum_{v \in Values(A)} \frac{|D^v|}{|D|} Entropy(D^v)$$

$|D^v|$ **- a partition of *D* with the value of attribute A = *v***



Decision tree learning

- **Greedy learning algorithm**:
  - Builds the tree in the top-down fashion
  - Gradually expands the leaves of the partially built tree

Algorithm sketch:

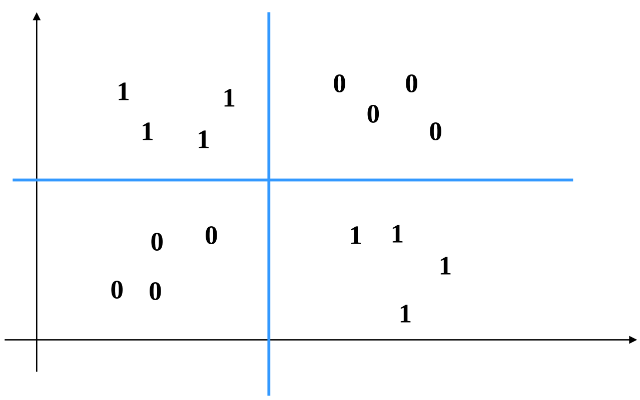
Repeat until no or small improvement in the impurity
- Find the attribute with the highest gain
- Add the attribute to the tree and split the set accordingly

The method is greedy:
- It looks at a single attribute and gain in each step
- May fail when the combination of attributes is needed to improve the purity (parity functions)

Decision tree learning

- **Limitations of greedy methods**

Cases in which only a combination of two or more attributes improves the impurity



Decision tree learning

By reducing the impurity measure we can grow **very large trees**

Problem: Overfitting

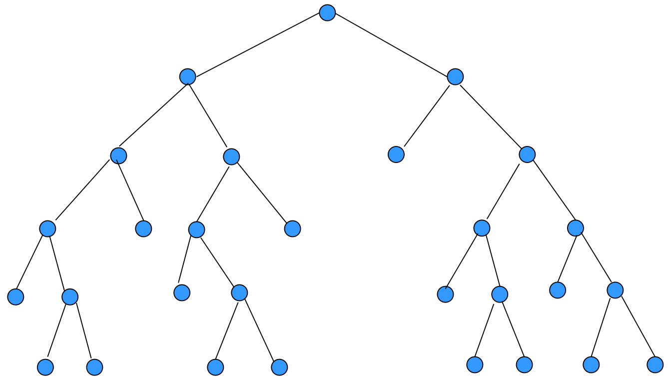
- We may split and classify very well the training set, but we may do worse in terms of the generalization error

Solutions to the overfitting problem:

- **Solution 1. Build the tree then prune the branches**
  - Build the tree, then eliminate leaves that overfit
  - Use **validation set** to test for the overfit
- **Solution 2. Prune while building the tree**
  - Test for the overfit in the tree building phase
  - Stop building the tree when performance on the **validation set** deteriorates

Decision tree learning

Backpruning: Prune branches of the tree built in the first phase in the botton-up fashion by using the validation set to test for the overfit



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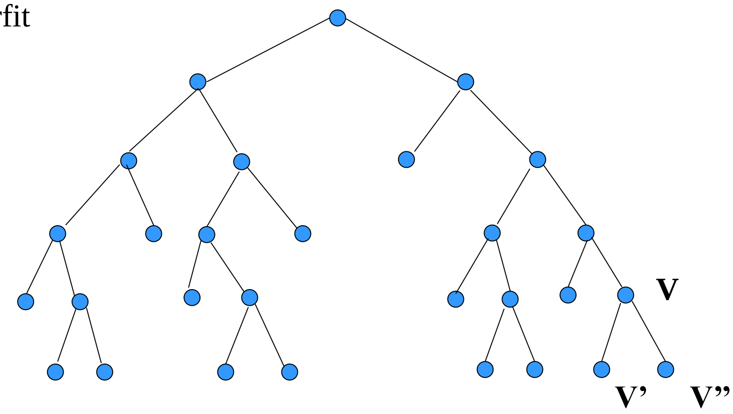

Compare: #Errors (V) vs #Error (V’) + # Errors(V”)

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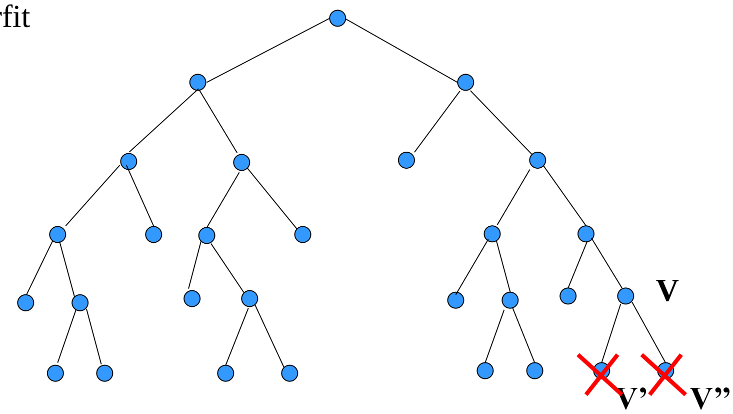

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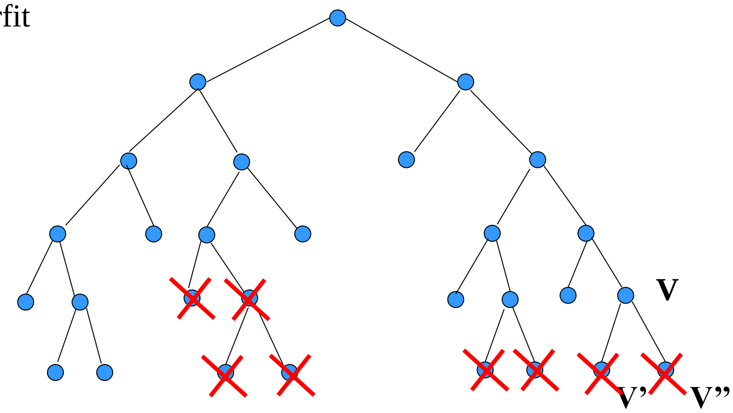

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