CS 2750 Machine Learning
Lecture 13

Bayesian belief networks

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Midterm exam

When: Wednesday, March 2, 2011

Midterm is:

- **In-class (75 minutes)**
- **closed book**
- **material covered during the semester including lecture today**

Project proposals

Due: **Wednesday, March 16, 2011**

- **1 page long**

Proposal

- **Written proposal:**
  1. Outline of a learning problem, type of data you have available. Why is the problem important?
  2. Learning methods you plan to try and implement for the problem. References to previous work.
  3. How do you plan to test, compare learning approaches
  4. Schedule of work (approximate timeline of work)

Bayesian belief networks (BBNs)

Bayesian belief networks.

- Represent the full joint distribution over the variables more compactly with a **smaller number of parameters**.
- Take advantage of **conditional and marginal independences** among random variables

- **A and B are independent**

$$P(A, B) = P(A)P(B)$$

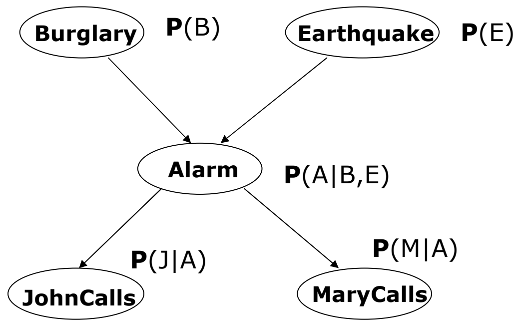
- **A and B are conditionally independent given C**

$$P(A, B \mid C) = P(A \mid C)P(B \mid C)$$

$$P(A \mid C, B) = P(A \mid C)$$

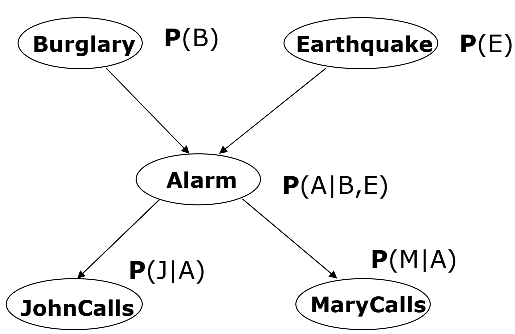
Bayesian belief network

1. **Directed acyclic graph**
   - **Nodes** = random variables
     Burglary, Earthquake, Alarm, Mary calls and John calls
   - **Links** = direct (causal) dependencies between variables.
     The chance of Alarm being is influenced by Earthquake,
     The chance of John calling is affected by the Alarm

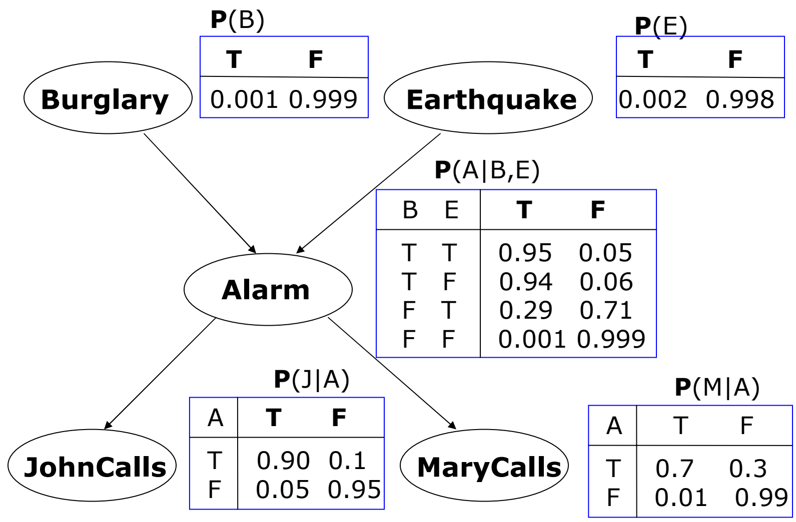


Bayesian belief network

2. **Local conditional distributions**
   - relate variables and their parents



Bayesian belief network

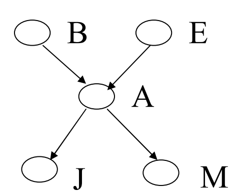


Full joint distribution in BBNs

Full joint distribution is defined in terms of local conditional distributions (obtained via the chain rule):

$$\mathbf{P}(X_1, X_2, .., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

Example:



Assume the following assignment of values to random variables

$B = T, E = T, A = T, J = T, M = F$

Then its probability is:

$$P(B = T, E = T, A = T, J = T, M = F) =$$

$$P(B = T)P(E = T)P(A = T \mid B = T, E = T)P(J = T \mid A = T)P(M = F \mid A = T)$$

Bayesian belief networks (BBNs)

Bayesian belief networks

- Represent the full joint distribution over the variables more compactly using the product of local conditionals.
- **But how did we get to local parameterizations?**

Answer:

- **Graphical structure** encodes **conditional and marginal independences** among random variables
- **A and B are independent** $P(A,B) = P(A)P(B)$
- **A and B are conditionally independent given C**

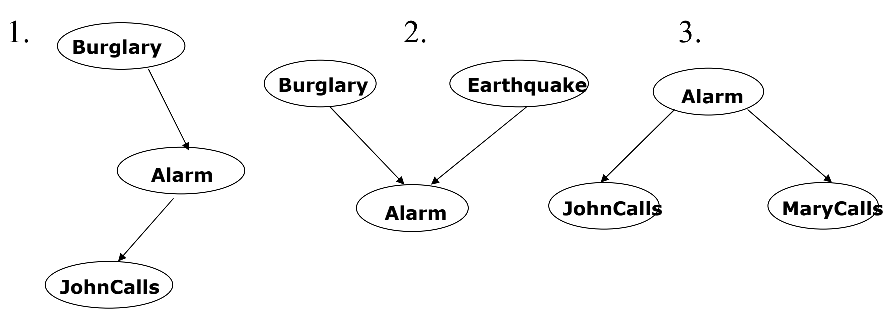
$$P(A \mid C, B) = P(A \mid C)$$

$$P(A, B \mid C) = P(A \mid C)P(B \mid C)$$

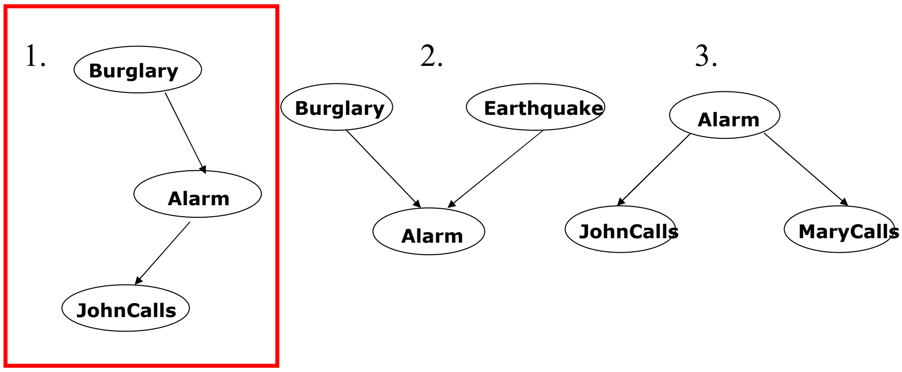
- **The graph structure implies the decomposition !!!**

Independences in BBNs

3 basic independence structures:



Independences in BBNs

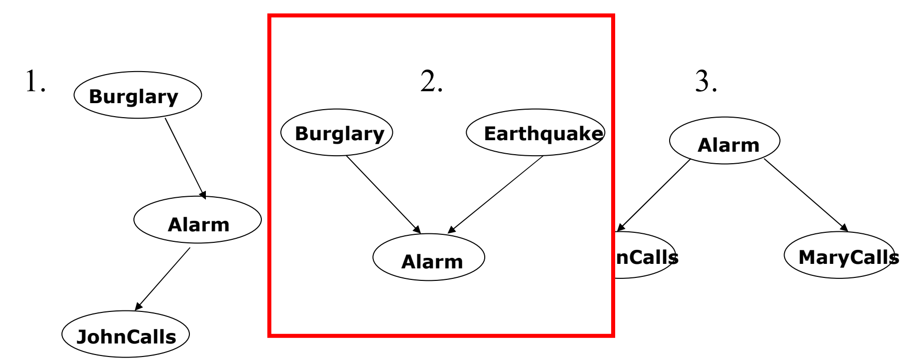


1. JohnCalls **is independent** of Burglary given Alarm

$$P(J \mid A, B) = P(J \mid A)$$

$$P(J, B \mid A) = P(J \mid A)P(B \mid A)$$

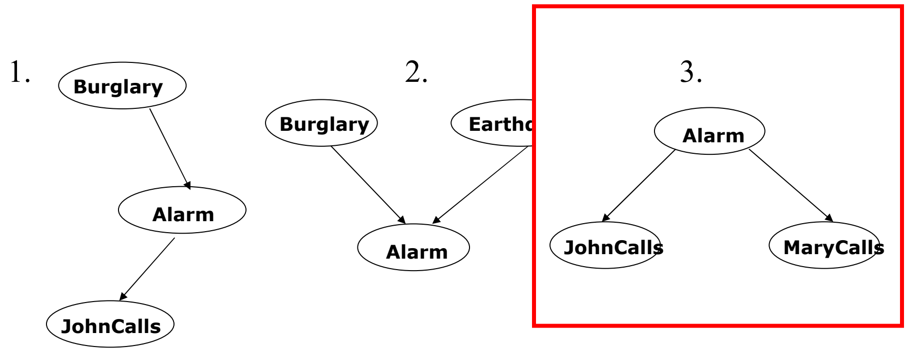
Independences in BBNs



2. Burglary **is independent** of Earthquake (not knowing Alarm)
   Burglary and Earthquake **become dependent** given Alarm !!

$$P(B, E) = P(B)P(E)$$

Independences in BBNs



3. MaryCalls **is independent** of JohnCalls given Alarm

$$P(J \mid A, M) = P(J \mid A)$$

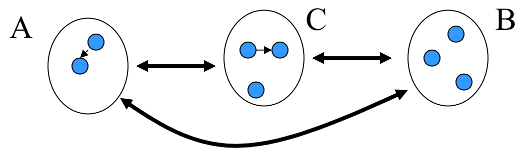
$$P(J, M \mid A) = P(J \mid A) P(M \mid A)$$

Independence in BBN

- BBN distribution models many conditional independence relations relating distant variables and sets
- These are defined in terms of the graphical criterion called d-separation
- **D-separation in the graph**
  - Let X,Y and Z be three sets of nodes
  - If X and Y are d-separated by Z then X and Y are conditionally independent given Z
- **D-separation :**
  - **A is d-separated from B given C** if every undirected path between them is **blocked**
- **Path blocking**
  - 3 cases that expand on three basic independence structures

Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**



Undirected path blocking

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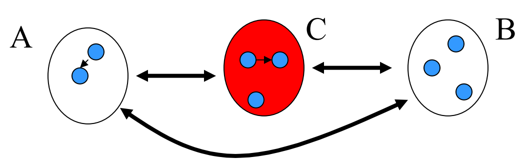
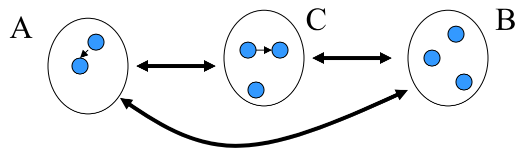
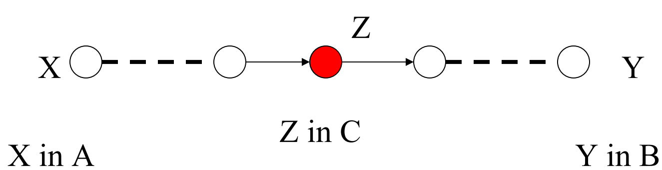


Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**



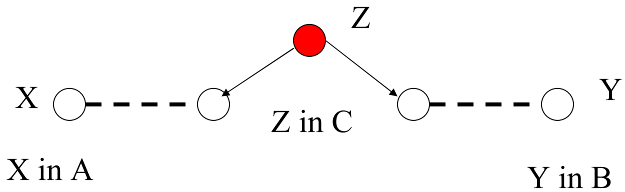
- **1. Path blocking with a linear substructure**



Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**

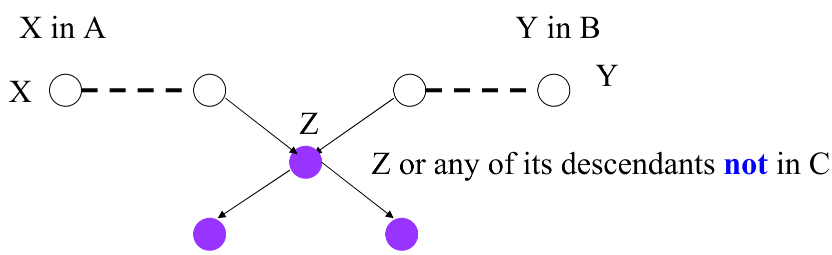
- **2. Path blocking with the wedge substructure**



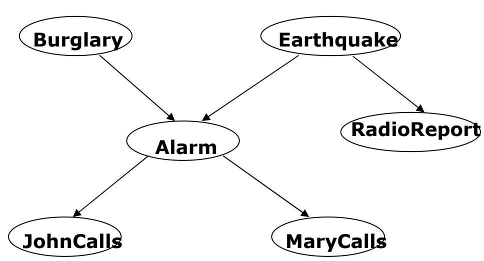
Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**

- **3. Path blocking with the vee substructure**

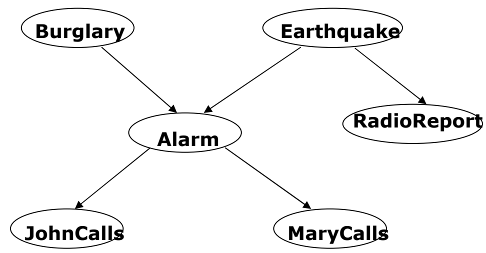


Independences in BBNs



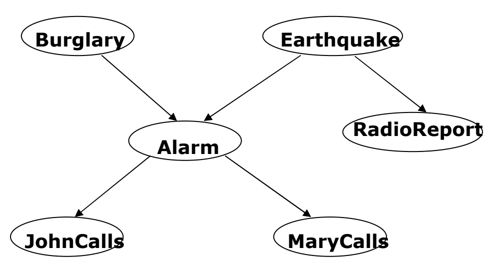
- Earthquake and Burglary are independent given MaryCalls **?**

Independences in BBNs



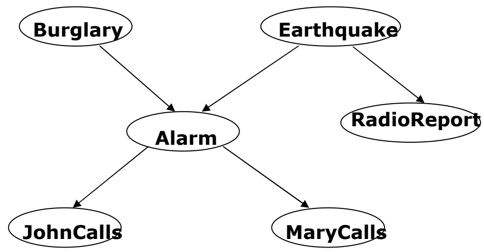
- Earthquake and Burglary are independent given MaryCalls **F**
- Burglary and MaryCalls are independent (not knowing Alarm) **?**

Independences in BBNs



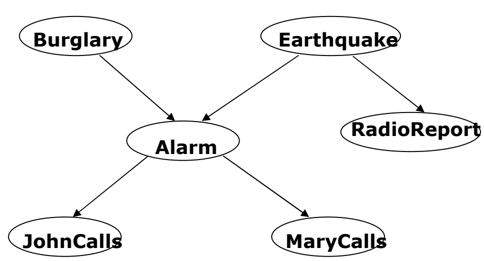
- Earthquake and Burglary are independent given MaryCalls **F**
- Burglary and MaryCalls are independent (not knowing Alarm) **F**
- Burglary and RadioReport are independent given Earthquake **?**

Independences in BBNs



- Earthquake and Burglary are independent given MaryCalls **F**
- Burglary and MaryCalls are independent (not knowing Alarm) **F**
- Burglary and RadioReport are independent given Earthquake **T**
- Burglary and RadioReport are independent given MaryCalls **?**

Independences in BBNs



- Earthquake and Burglary are independent given MaryCalls **F**
- Burglary and MaryCalls are independent (not knowing Alarm) **F**
- Burglary and RadioReport are independent given Earthquake **T**
- Burglary and RadioReport are independent given MaryCalls **F**

Bayesian belief networks (BBNs)

Bayesian belief networks

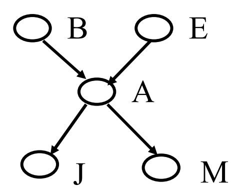
- Represents the full joint distribution over the variables more compactly using the product of local conditionals.
- **So how did we get to local parameterizations?**

$$\mathbf{P}(X_1, X_2, .., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

- **The decomposition is implied by the set of independences encoded in the belief network.**

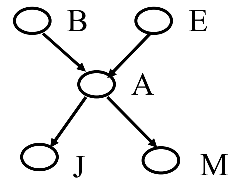
Full joint distribution in BBNs

Rewrite the full joint probability using the product rule:



$$P(B=T, E=T, A=T, J=T, M=F) =$$

Full joint distribution in BBNs

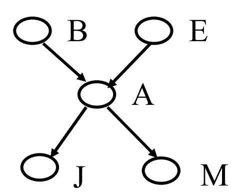


Rewrite the full joint probability using the product rule:

$$P(B=T,E=T,A=T,J=T,M=F)=$$

$$=P(J=T \mid B=T,E=T,A=T,M=F)P(B=T,E=T,A=T,M=F)$$
$$=\underline{P(J=T \mid A=T)}P(B=T,E=T,A=T,M=F)$$

Full joint distribution in BBNs



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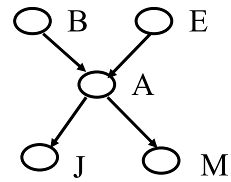
$$=\underline{P(J=T \mid A=T)}P(B=T,E=T,A=T,M=F)$$
$$P(M=F \mid B=T,E=T,A=T)P(B=T,E=T,A=T)$$
$$\underline{P(M=F \mid A=T)}P(B=T,E=T,A=T)$$

Full joint distribution in BBNs



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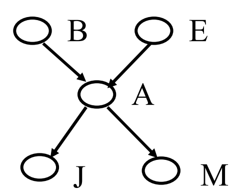
$$=\underline{P(J=T\mid A=T)}P(B=T,E=T,A=T,M=F)$$

$$P(M=F\mid B=T,E=T,A=T)P(B=T,E=T,A=T)$$

$$\underline{P(M=F\mid A=T)}P(B=T,E=T,A=T)$$

$$\underline{P(A=T\mid B=T,E=T)}P(B=T,E=T)$$

Full joint distribution in BBNs



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$$P(M=F\mid B=T,E=T,A=T)P(B=T,E=T,A=T)$$

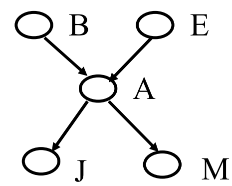
$$\underline{P(M=F\mid A=T)}P(B=T,E=T,A=T)$$

$$\underline{P(A=T\mid B=T,E=T)}P(B=T,E=T)$$

$$P(B=T)P(E=T)$$

Full joint distribution in BBNs

Rewrite the full joint probability using the product rule:



$$P(B=T,E=T,A=T,J=T,M=F)=$$

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$$P(M=F\mid A=T)P(B=T,E=T,A=T)$$

$$P(A=T\mid B=T,E=T)P(B=T,E=T)$$

$$P(B=T)P(E=T)$$

$$=P(J=T\mid A=T)P(M=F\mid A=T)P(A=T\mid B=T,E=T)P(B=T)P(E=T)$$

Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$\mathbf{P}(X_1, X_2, \ldots, X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

- **What did we save?**

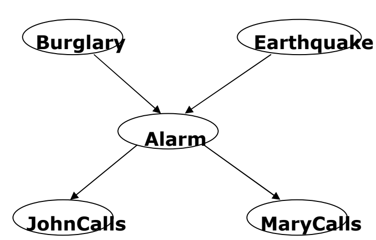
Alarm example: 5 binary (True, False) variables

# of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

$$2^5 - 1 = 31$$



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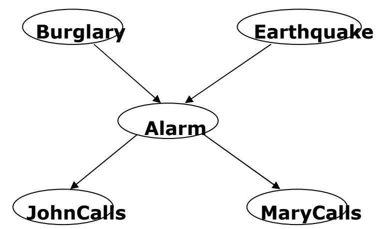
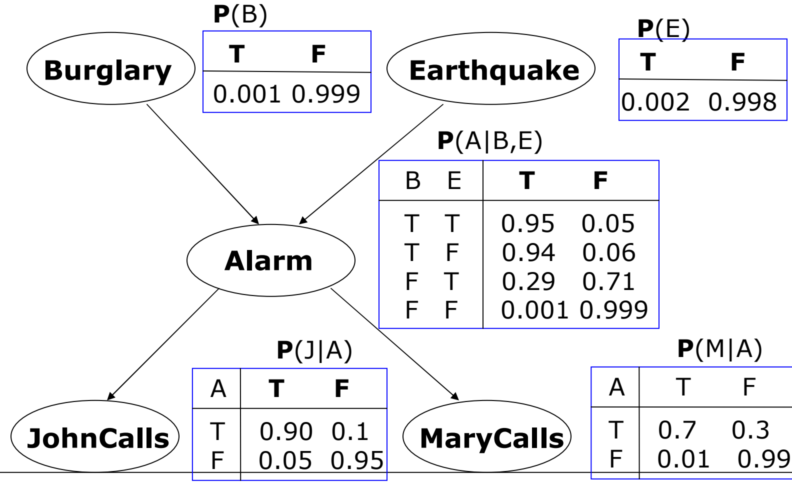
# of parameters of the BBN: ?



Bayesian belief network.

- In the BBN the **full joint distribution** is expressed using a set of local conditional distributions



P(B)

T	F
0.001	0.999

P(E)

T	F
0.002	0.998

P(A|B,E)

B	E	T	F
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

P(J|A)

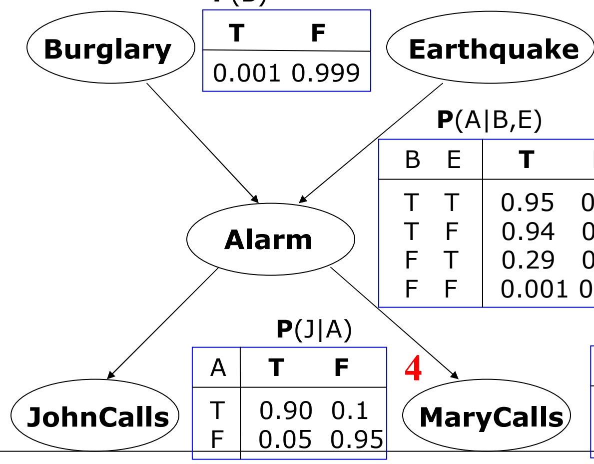
A	T	F
T	0.90	0.1
F	0.05	0.95

P(M|A)

A	T	F
T	0.7	0.3
F	0.01	0.99

Bayesian belief network.

- In the BBN the **full joint distribution** is expressed using a set of local conditional distributions



P(B) — 2

T	**F**
0.001	0.999

P(E) — 2

T	**F**
0.002	0.998

P(A|B,E) — 8

B	E	**T**	**F**
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

P(J|A) — 4

A	**T**	**F**
T	0.90	0.1
F	0.05	0.95

P(M|A) — 4

A	T	F
T	0.7	0.3
F	0.01	0.99

Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$\mathbf{P}(X_1, X_2, .., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

- **What did we save?**

Alarm example: 5 binary (True, False) variables

# of parameters of the full joint:

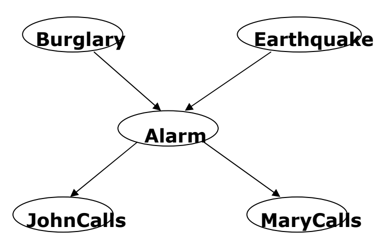
$$2^5 = 32$$

One parameter is for free:

$$2^5 - 1 = 31$$

# of parameters of the BBN:

$$2^3 + 2(2^2) + 2(2) = 20$$



One parameter in every conditional is for free:

?

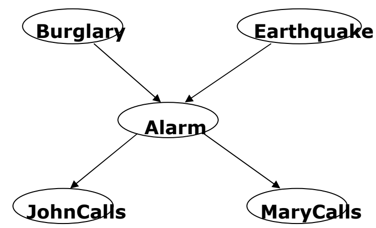
Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$\mathbf{P}(X_1, X_2, .., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

- **What did we save?**

Alarm example: 5 binary (True, False) variables



# of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

$$2^5 - 1 = 31$$

# of parameters of the BBN:

$$2^3 + 2(2^2) + 2(2) = 20$$

One parameter in every conditional is for free:

$$2^2 + 2(2) + 2(1) = 10$$

Model acquisition problem

The structure of the BBN typically reflects causal relations

- BBNs are also sometime referred to as **causal networks**
- Causal structure is very intuitive in many applications domain and it is relatively easy to obtain from the domain expert

Probability parameters of BBN correspond to conditional distributions relating a random variable and its parents only

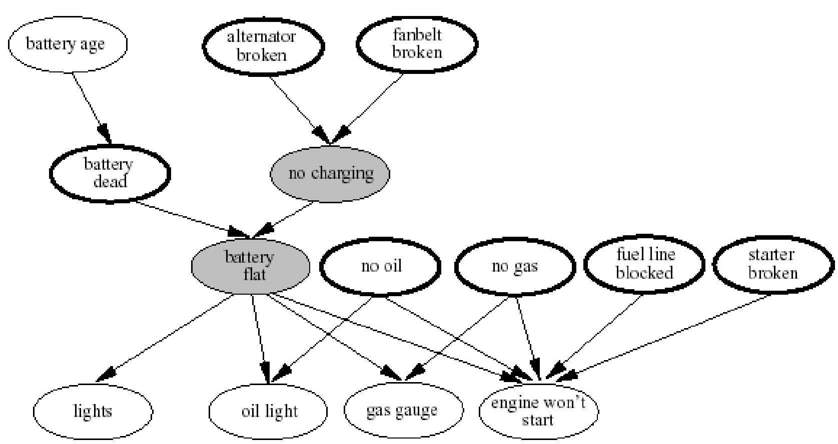
- Their complexity much smaller than the full joint
- Easier to come up (estimate) the probabilities from expert or automatically by learning from data

BBNs built in practice

- **In various areas:**
  - Intelligent user interfaces (Microsoft)
  - Troubleshooting, diagnosis of a technical device
  - Medical diagnosis:
    - Pathfinder (Intellipath)
    - CPSC
    - Munin
    - QMR-DT
  - Collaborative filtering
  - Military applications
  - Insurance, credit applications

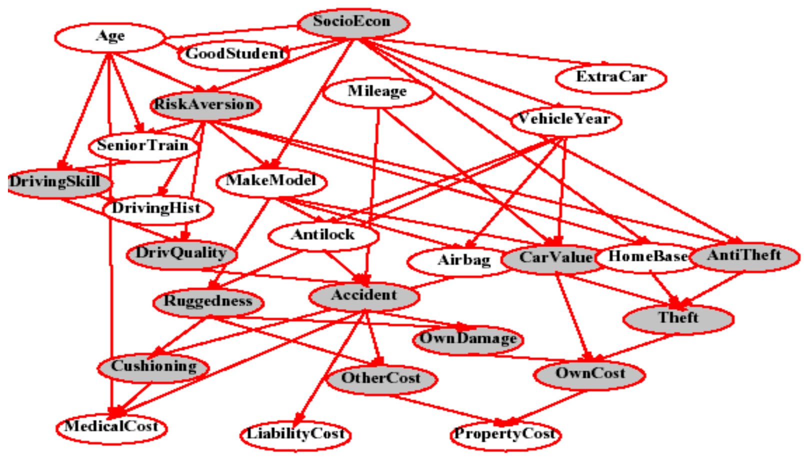
Diagnosis of car engine

- Diagnose the engine start problem

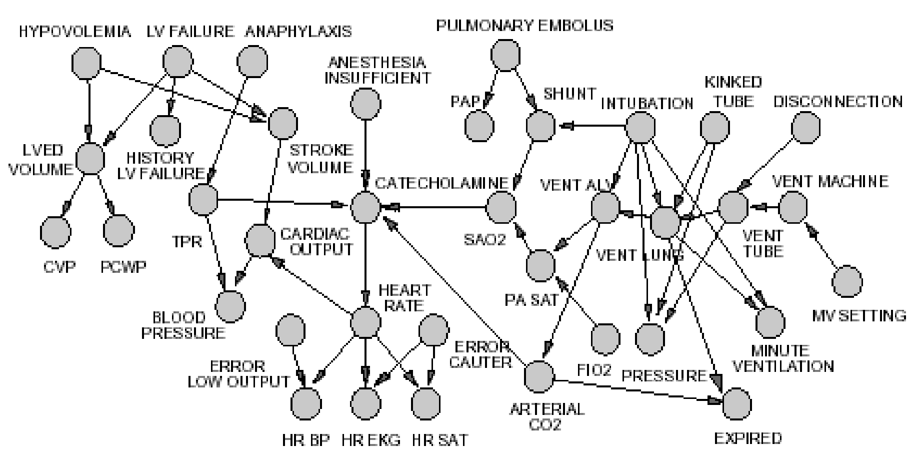


Car insurance example

- Predict claim costs (medical, liability) based on application data

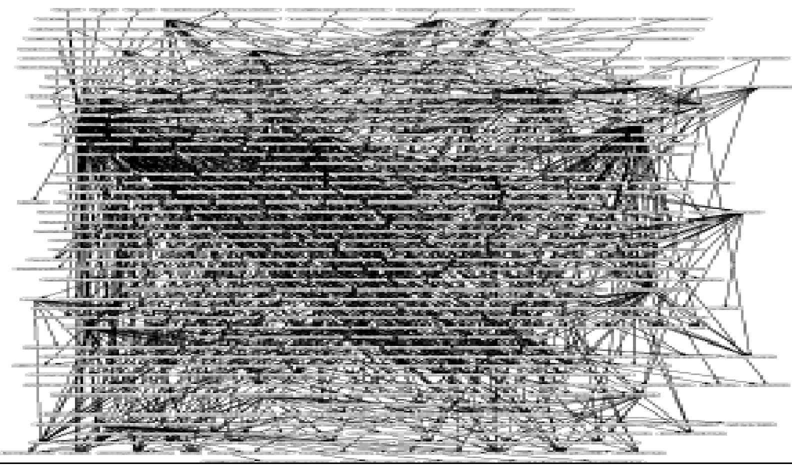


(ICU) Alarm network



CPCS

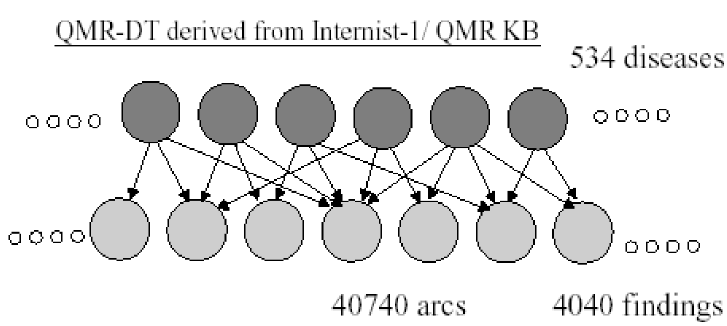
- **C**omputer-based **P**atient **C**ase **S**imulation system (CPCS-PM) developed by Parker and Miller (at University of Pittsburgh)
- 422 nodes and 867 arcs



QMR-DT

- **Medical diagnosis in internal medicine**

  Bipartite network of disease/findings relations



Inference in Bayesian networks

- BBN models compactly the full joint distribution by taking advantage of existing independences between variables
- Simplifies the acquisition of a probabilistic model
- But we are interested in solving various **inference tasks**:
  - **Diagnostic task. (from effect to cause)**

    $$\mathbf{P}(Burglary \mid JohnCalls = T)$$

  - **Prediction task. (from cause to effect)**

    $$\mathbf{P}(JohnCalls \mid Burglary = T)$$

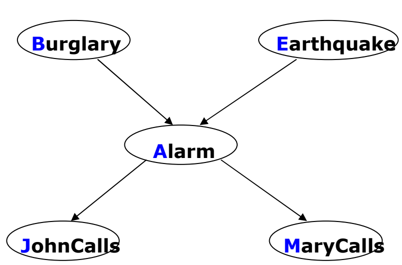
  - **Other probabilistic queries** (queries on joint distributions).

    $$\mathbf{P}(Alarm)$$

- **Main issue:** Can we take advantage of independences to construct special algorithms and speeding up the inference?

Inference in Bayesian network

- **Bad news:**
  - Exact inference problem in BBNs is NP-hard (Cooper)
  - Approximate inference is NP-hard (Dagum, Luby)
- **But** very often we can achieve significant improvements
- Assume our Alarm network



- Assume we want to compute: $P(J = T)$

Inference in Bayesian networks

Computing: $P(J = T)$

Approach 1. Blind approach.

- Sum out all un-instantiated variables from the full joint,
- express the joint distribution as a product of conditionals

$$P(J = T) =$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(B = b, E = e, A = a, J = T, M = m)$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(A = a \mid B = b, E = e) P(B = b) P(E = e)$$

Computational cost:

Number of additions: ?

Number of products: ?

Inference in Bayesian networks

Computing: $P(J = T)$

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Computational cost:

Number of additions: 15

Number of products: ?

Inference in Bayesian networks

Computing: $P(J = T)$

Approach 1. Blind approach.

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- express the joint distribution as a product of conditionals

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$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(A = a \mid B = b, E = e) P(B = b) P(E = e)$$

Computational cost:

Number of additions: 15

Number of products: 16*4=64

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J = T) =$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(A = a \mid B = b, E = e) P(B = b) P(E = e)$$

$$= \sum_{b \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(B = b) [\sum_{e \in T,F} P(A = a \mid B = b, E = e) P(E = e)]$$

$$= \sum_{a \in T,F} P(J = T \mid A = a) [\sum_{m \in T,F} P(M = m \mid A = a)] [\sum_{b \in T,F} P(B = b) [\sum_{e \in T,F} P(A = a \mid B = b, E = e) P(E = e)]]$$

Computational cost:

Number of additions: 1+2*[1+1+2*1]=**?**

Number of products: 2*[2+2*(1+2*1)]=?

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J=T)=$$

$$=\sum_{b\in T,F}\sum_{e\in T,F}\sum_{a\in T,F}\sum_{m\in T,F}P(J=T\mid A=a)P(M=m\mid A=a)P(A=a\mid B=b,E=e)P(B=b)P(E=e)$$

$$=\sum_{b\in T,F}\sum_{a\in T.F}\sum_{m\in T,F}P(J=T\mid A=a)P(M=m\mid A=a)P(B=b)[\sum_{e\in T,F}P(A=a\mid B=b,E=e)P(E=e)]$$

$$=\sum_{a\in T,F}P(J=T\mid A=a)[\sum_{m\in T,F}P(M=m\mid A=a)][\sum_{b\in T,F}P(B=b)[\sum_{e\in T,F}P(A=a\mid B=b,E=e)P(E=e)]]$$

Computational cost:

Number of additions: 1+2*[1+1+2*1]=**9**

Number of products: 2*[2+2*(1+2*1)]=?

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J=T)=$$

$$=\sum_{b\in T,F}\sum_{e\in T,F}\sum_{a\in T,F}\sum_{m\in T,F}P(J=T\mid A=a)P(M=m\mid A=a)P(A=a\mid B=b,E=e)P(B=b)P(E=e)$$

$$=\sum_{b\in T,F}\sum_{a\in T.F}\sum_{m\in T,F}P(J=T\mid A=a)P(M=m\mid A=a)P(B=b)[\sum_{e\in T,F}P(A=a\mid B=b,E=e)P(E=e)]$$

$$=\sum_{a\in T,F}P(J=T\mid A=a)[\sum_{m\in T,F}P(M=m\mid A=a)][\sum_{b\in T,F}P(B=b)[\sum_{e\in T,F}P(A=a\mid B=b,E=e)P(E=e)]]$$

Computational cost:

Number of additions: 1+2*[1+1+2*1]=**9**

Number of products: 2*[2+2*(1+2*1)]=16

Inference in Bayesian networks

- When cashing of results becomes handy?
- What if we want to compute a diagnostic query:

$$P(B=T \mid J=T) = \frac{P(B=T, J=T)}{P(J=T)}$$

- Exactly probabilities we have just compared !!
- There are other queries when cashing and ordering of sums and products can be shared and saves computation

$$\mathbf{P}(B \mid J=T) = \frac{\mathbf{P}(B, J=T)}{P(J=T)} = \alpha \mathbf{P}(B, J=T)$$

- General technique: **Recursive decomposition**

Variable elimination

- **Recursive decomposition:**
  – Interleave sum and products before inference
- **Variable elimination:**
  – Similar idea but interleave sum and products one variable at the time during inference
  – E.g. Query $P(J=T)$ requires to eliminate A,B,E,M and this can be done in different order

$$P(J=T) =$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J=T \mid A=a) P(M=m \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e)$$

Variable elimination

Assume order: M, E, B,A to calculate $P(J=T)$

$$= \sum_{b\in T,F} \sum_{e\in T,F} \sum_{a\in T,F} \sum_{m\in T,F} P(J=T \mid A=a) P(M=m \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e)$$

$$= \sum_{b\in T,F} \sum_{e\in T,F} \sum_{a\in T,F} P(J=T \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e) \left[\sum_{m\in T,F} P(M=m \mid A=a) \right]$$

$$= \sum_{b\in T,F} \sum_{e\in T,F} \sum_{a\in T,F} P(J=T \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e) \quad 1$$

$$= \sum_{a\in T,F} \sum_{b\in T,F} P(J=T \mid A=a) P(B=b) \left[\sum_{e\in T,F} P(A=a \mid B=b, E=e) P(E=e) \right]$$

$$= \sum_{a\in T,F} \sum_{b\in T,F} P(J=T \mid A=a) P(B=b)\, \tau_1(A=a, B=b)$$

$$= \sum_{a\in T,F} P(J=T \mid A=a) \left[\sum_{e\in T,F} P(B=b)\, \tau_1(A=a, B=b) \right]$$

$$= \sum_{a\in T,F} P(J=T \mid A=a) \quad \tau_2(A=a)$$

Inference in Bayesian network

- **Exact inference algorithms:**
  - Book → **Variable elimination**
  - **Recursive decomposition** (Cooper, Darwiche)
  - Symbolic inference (D'Ambrosio)
  - Belief propagation algorithm (Pearl)
  - Book → **Clustering and joint tree approach (Lauritzen, Spiegelhalter)**
  - Arc reversal (Olmsted, Schachter)
- **Approximate inference algorithms:**
  - Book → **Monte Carlo methods:**
    - Forward sampling, Likelihood sampling
  - Variational methods

Learning of BBN

Learning.

- **Learning of parameters of conditional probabilities**
- **Learning of the network structure**

Variables:

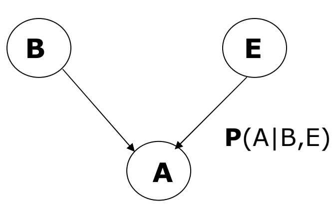
- **Observable** – values present in every data sample
- **Hidden** – they values are never observed in data
- **Missing values** – values sometimes present, sometimes not

Next:

- Learning of parameters of BBN
- All variables are observable

Estimation of parameters of BBN

- **Idea:** decompose the estimation problem for the full joint over a large number of variables to a set of smaller estimation problems corresponding to local parent-variable conditionals.
- **Example:** Assume A,E,B are binary with *True, False* values



4 estimation problems

- **P**(A|B=T,E=T)
- **P**(A|B=T,E=F)
- **P**(A|B=F,E=T)
- **P**(A|B=F,E=F)

- **Assumption that enables the decomposition:** parameters of conditional distributions are independent

Estimates of parameters of BBN

- Two assumptions that permit the decomposition:
  - **Sample independence**

$$P(D \mid \mathbf{\Theta}, \xi) = \prod_{u=1}^{N} P(D_u \mid \mathbf{\Theta}, \xi)$$

  - **Parameter independence**

# of nodes (n)
# of parents values (q_i)

$$p(\mathbf{\Theta} \mid D, \xi) = \prod_{i=1}^{n} \prod_{j=1}^{q_i} p(\theta_{ij} \mid D, \xi)$$

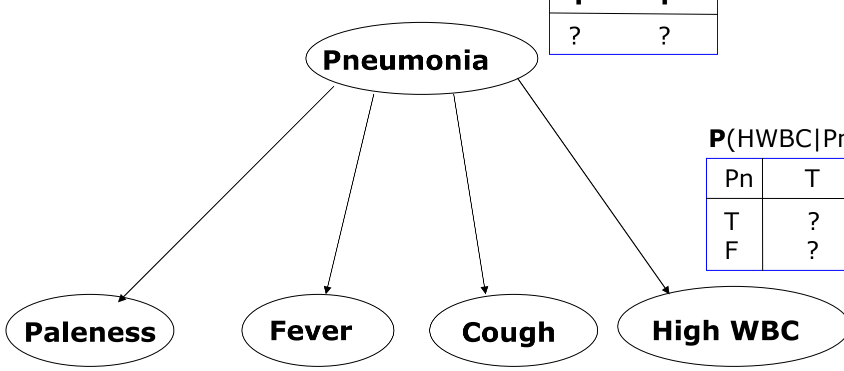
Parameters of **each conditional** (one for every assignment of values to parent variables) can be learned independently

Learning of BBN parameters. Example.

Example:

P(Pneumonia)

T	F
?	?



P(HWBC|Pneum)

Pn	T	F
T	?	?
F	?	?

P(Palen|Pneum)
?

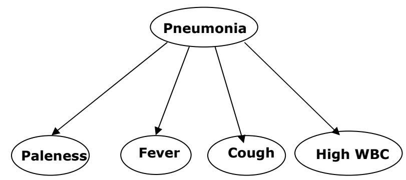
P(Fever|Pneum)
?

P(Cough|Pneum)
?

Learning of BBN parameters. Example.

Data D (different patient cases):

Pal	Fev	Cou	HWB	Pneu
T	T	T	T	F
T	F	F	F	F
F	F	T	T	T
F	F	T	F	T
F	T	T	T	T
T	F	T	F	F
F	F	F	F	F
T	T	F	F	F
T	T	T	T	T
F	T	F	T	T
T	F	F	T	F
F	T	F	F	F



Estimates of parameters of BBN

- Much like multiple **coin toss or roll of a dice** problems.
- A "smaller" learning problem corresponds to the learning of exactly one conditional distribution
- **Example:**

$$\mathbf{P}(Fever \mid Pneumonia = T)$$

- **Problem:** How to pick the data to learn?

Estimates of parameters of BBN

Much like multiple **coin toss or roll of a dice** problems.

- A "smaller" learning problem corresponds to the learning of exactly one conditional distribution

Example:

$$\mathbf{P}(Fever \mid Pneumonia = T)$$

Problem: How to pick the data to learn?

Answer:

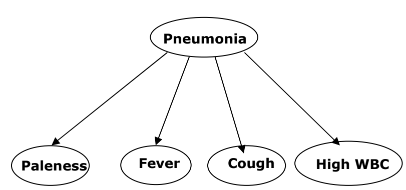
1. Select data points with Pneumonia=T (ignore the rest)
2. Focus on (select) only values of the random variable defining the distribution (Fever)
3. Learn the parameters of the conditional the same way as we learned the parameters for a coin or a dice

Learning of BBN parameters. Example.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 1: Select data points with Pneumonia=T

Pal	Fev	Cou	HWB	Pneu
T	T	T	T	F
T	F	F	F	F
F	F	T	T	T
F	F	T	F	T
F	T	T	T	T
T	F	T	F	F
F	F	F	F	F
T	T	F	F	F
T	T	T	T	T
F	T	F	T	T
T	F	F	T	F
F	T	F	F	F

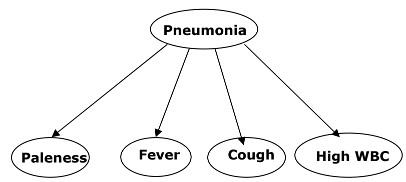


Learning of BBN parameters. Example.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 1: Ignore the rest

Pal	Fev	Cou	HWB	Pneu
F	F	T	T	T
F	F	T	F	T
F	T	T	T	T
T	T	T	T	T
F	T	F	T	T

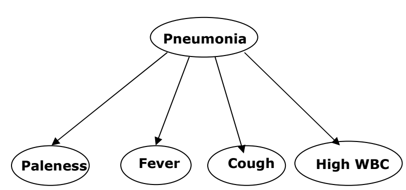


Learning of BBN parameters. Example.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 2: Select values of the random variable defining the distribution of Fever

Pal	Fev	Cou	HWB	Pneu
F	F	T	T	T
F	F	T	F	T
F	T	T	T	T
T	T	T	T	T
F	T	F	T	T

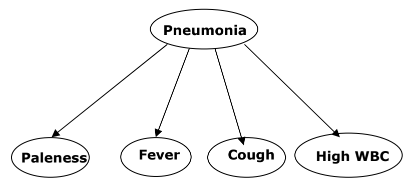


Learning of BBN parameters. Example.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 2: Ignore the rest

Fev
F
F
T
T
T

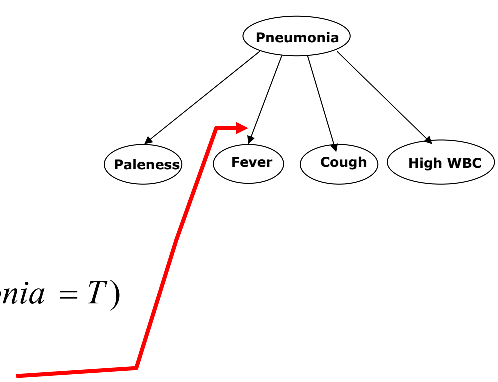


Learning of BBN parameters. Example.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 3a: Learning the ML estimate

Fev
F
F
T
T
T



$\mathbf{P}(Fever \mid Pneumonia = T)$

T	F
0.6	0.4

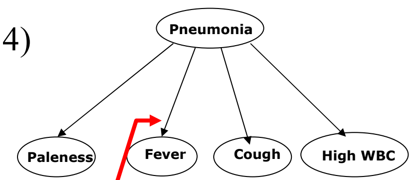
Learning of BBN parameters. Bayesian learning.

Learn: $\mathbf{P}(Fever \mid Pneumonia = T)$

Step 3b: Learning the Bayesian estimate

Assume the prior

$$\theta_{Fever|Pneumonia=T} \sim Beta(3,4)$$



Fev
F
F
T
T
T

Posterior:

$$\theta_{Fever|Pneumonia=T} \sim Beta(6,6)$$