CS 2750 Machine Learning
Lecture 12

Naïve Bayes classifier & Evaluation framework

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Generative approach to classification

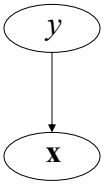
Idea:

1. **Represent and learn the distribution** $p(\mathbf{x}, y)$
2. **Use it to define probabilistic discriminant functions**

E.g. $g_o(\mathbf{x}) = p(y = 0 \mid \mathbf{x})$ $g_1(\mathbf{x}) = p(y = 1 \mid \mathbf{x})$

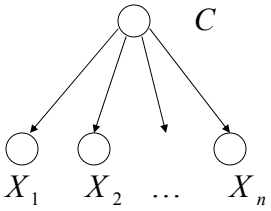
Typical model $p(\mathbf{x}, y) = p(\mathbf{x} \mid y)p(y)$

- $p(\mathbf{x} \mid y)$ = **Class-conditional distributions (densities)**
  binary classification: two class conditional distributions
  $p(\mathbf{x} \mid y = 0)$ $p(\mathbf{x} \mid y = 1)$
- $p(y)$ = **Priors on classes** - probability of class y
  binary classification: Bernoulli distribution
  $p(y = 0) + p(y = 1) = 1$



Naïve Bayes classifier

- **a generative classifier model with an additional simplifying assumption:**
  - All input attributes are conditionally independent of each other given the class. So we have:



$$p(\mathbf{x}, y) = p(\mathbf{x} \mid y)\, p(y)$$

$$p(\mathbf{x} \mid y) = \prod_{i=1}^{N} p(x_i \mid y)$$

Learning of parameters of the model

Much simpler density estimation problems

- We need to learn:

  $p(\mathbf{x} \mid y = 0)$ and $p(\mathbf{x} \mid y = 1)$ and $p(y)$

- Because of the assumption of the conditional independence we need to learn:

  for every variable i: $p(x_i \mid y = 0)$ and $p(x_i \mid y = 1)$

- **If the number of input attributes is large this much easier**
- **Also, the model gives us a flexibility to represent input attributes different of different forms !!!**
- E.g. one attribute can be modeled using the Bernoulli, the other as Gaussian density, or as a Poisson distribution

Making a class decision for the Naïve Bayes

Discriminant functions.

- **Likelihood of data** – choose the class that explains the input data (**x**) better (likelihood of the data)

$$\underbrace{\prod_{i=1}^{N} p(x_i \mid \Theta_{1,i})}_{g_1(\mathbf{x})} > \underbrace{\prod_{i=1}^{N} p(x_i \mid \Theta_{2,i})}_{g_0(\mathbf{x})} \Rightarrow \begin{array}{l}\text{then } y=1 \\ \text{else } y=0\end{array}$$

- **Posterior of a class** – choose the class with better posterior probability $p(y=1 \mid \mathbf{x}) > p(y=0 \mid \mathbf{x})$

then $y=1$
else $y=0$

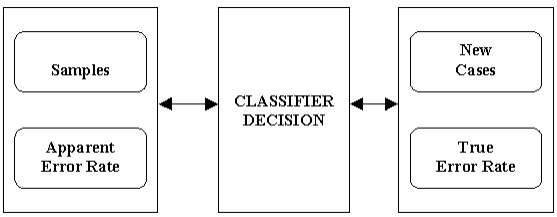
$$p(y=1 \mid \mathbf{x}) = \frac{\left(\prod_{i=1}^{N} p(x_i \mid \Theta_{1,i})\right) p(y=1)}{\left(\prod_{i=1}^{N} p(x_i \mid \Theta_{1,i})\right) p(y=0) + \left(\prod_{i=1}^{N} p(x_i \mid \Theta_{2,i})\right) p(y=1)}$$

Experimental evaluation

Dataset: a set of samples

Split the dataset to: **Training and testing data**

- Learn on the Training data
- Test on the Testing data
- Test errors give an honest assesment of the error for future cases (recall the overfit issue)



Prevent the train/test split bias

If we use only one train/test split we can be lucky or unlucky

A much better (less biased) option is to use multiple train/test splits and average the test errors obtained on these splits

How to do the splits ?

- **Random subsampling:** choose the test and train set randomly k times
- **Cross-fold validation:** a more systematic approach
  - Split data to k equal partitions
  - Create a train data using k lpartitions, test data on the remaining partition
  - Gives us k different train test splits

Evaluation

For any data set we used to test the model we can build a confusion matrix:

- Counts of examples with:
- class label ω_j that are classified with a label α_i

		model	
		$\alpha = 1$	$\alpha = 0$
target	$\omega = 1$	140	20
	$\omega = 0$	17	54

Evaluation

For any data set we used to test the model we can build a confusion matrix:

model

target		$\alpha = 1$	$\alpha = 0$
	$\omega = 1$	140	20
	$\omega = 0$	17	54

agreement

Error: ?

Evaluation for the binary classification

For any data set we used to test the model we can build a confusion matrix:

model

target		$\alpha = 1$	$\alpha = 0$
	$\omega = 1$	*TP*	*FN*
	$\omega = 0$	*FP*	*TN*

TP: True positive (hit)
FP: False positive (false alarm)
TN: True negative (correct rejection)
FN: False negative (a miss)

Additional statistics

- **Sensitivity**

$$SENS = \frac{TP}{TP + FN}$$

- **Specificity**

$$SPEC = \frac{TN}{TN + FP}$$

- **Positive predictive value**

$$PPT = \frac{TP}{TP + FP}$$

- **Negative predictive value**

$$NPV = \frac{TN}{TN + FN}$$

Binary classification. Additional quantities.

- **Confusion matrix**

		model: 1	model: 0	
target	1	140	20	$SENS = 140/160$
	0	10	180	$SPEC = 180/190$
		$PPV = 140/150$	$NPV = 180/200$	

Row and column quantities:

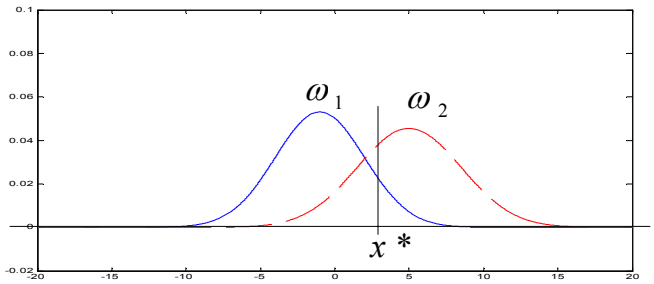
- Sensitivity (SENS)
- Specificity (SPEC)
- Positive predictive value (PPV)
- Negative predictive value (NPV)

Receiver operating characteristic

ROC

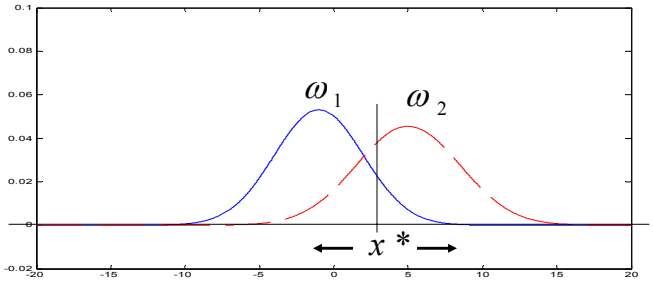
- shows the discriminability between the two classes under different decision biases (types of errors we make matter)
- ROC curve is created by plotting:
- **the true positive rate against false positive rates**
- or **sensitivity against (1-specificity)**

Binary decisions: accuracy.



- **Probabilities:**
  - *True positive (hit)* $p(x > x^* \mid \mathbf{x} \in \omega_2)$
  - *False positive (false alarm)* $p(x > x^* \mid \mathbf{x} \in \omega_1)$
  - *True negative (correct rejection)* $p(x < x^* \mid \mathbf{x} \in \omega_1)$
  - *False negative (a miss)* $p(x < x^* \mid \mathbf{x} \in \omega_2)$

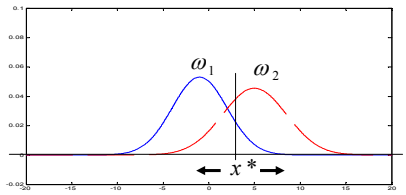
Decision threshold

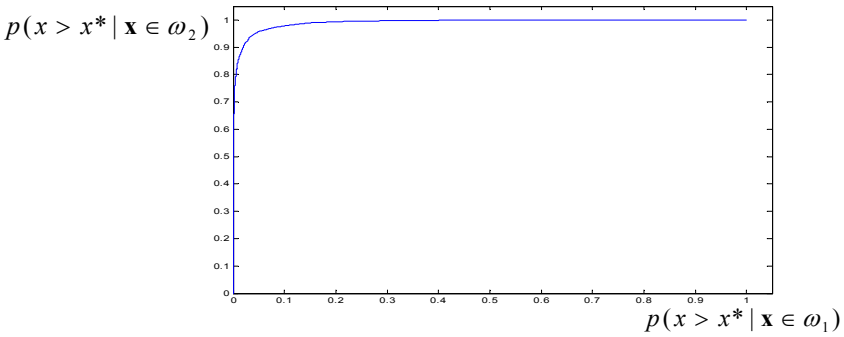


- **Movement of x* changes the probabilities:**
  - *True positive (hit)* $p(x > x^* \mid \mathbf{x} \in \omega_2)$
  - *False positive (false alarm)* $p(x > x^* \mid \mathbf{x} \in \omega_1)$
  - *True negative (correct rejection)* $p(x < x^* \mid \mathbf{x} \in \omega_1)$
  - *False negative (a miss)* $p(x < x^* \mid \mathbf{x} \in \omega_2)$

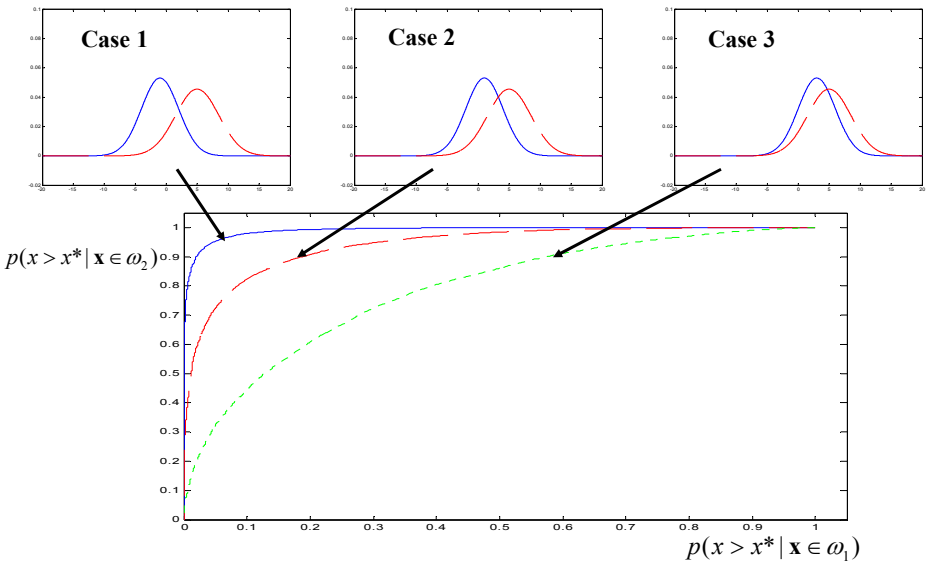
Receiver Operating Characteristic (ROC)

- **ROC curve plots :**
  $p(x > x^* \mid \mathbf{x} \in \omega_1)$
  vs $p(x > x^* \mid \mathbf{x} \in \omega_2)$
  for different x*





ROC curve



Bayesian decision theory

- **Assume we want to incorporate our bias about the learning into the learning process**
- **Assume a multiway classification problem and more general confusion matrix**
  - Counts of examples with:
  - class label ω_j that are classified with a label α_i

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
$\omega = 0$	140	20	22
$\omega = 1$	17	54	8
$\omega = 2$	12	4	76

agreement

Zero-one loss function

- **Misclassification error**
  - Based on the zero one loss function
    - Any misclassified example counts as 1
    - Correctly classified example counts as 0

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
$\omega = 0$	140	20	22
$\omega = 1$	17	54	8
$\omega = 2$	12	4	76

agreement

- What is the zero one loss for the confusion matrix?

General loss function

- **Error function based on a more general loss function**
  - Different misclassifications have different weight (loss)
  - α_i our choice
  - ω_j true label
  - $\lambda(\alpha_i \mid \omega_j)$ loss for classification

Example:

$\lambda(\alpha_i \mid \omega_j)$

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
$\omega = 0$	0	1	5
$\omega = 1$	3	0	2
$\omega = 2$	3	1	0

Bayesian decision theory

- **More general loss function**
  - Different misclassifications have different weight (loss)

    $\lambda(\alpha_i \mid \omega_j)$
- **Expected loss for choice** (action) α_i

$$R(\alpha_i \mid \mathbf{x}) = \sum_j \lambda(\alpha_i \mid \omega_j) P(\omega_j \mid \mathbf{x})$$

  - Also called conditional risk
- **Decision rule:** $\alpha(\mathbf{x})$
  - Chooses label (action) according to the input
- **Overall expected loss for the decision rule**

$$R(\alpha) = \int R(\alpha(\mathbf{x}), \mathbf{x}) P(\mathbf{x}) d\mathbf{x}$$

Bayesian decision theory

- **The optimal decision rule**

$$\alpha^*(\mathbf{x}) = \arg\max_{\alpha_i} \sum_j \lambda(\alpha_i \mid \omega_j) P(\omega_j \mid \mathbf{x})$$

How to modify classifiers to handle different loss?

- **Discriminative models:**
  - Directly optimize the parameters according to the new loss function
- **Generative models:**
  - Learn probabilities as before
  - Decisions about classes are biased to minimize the empirical loss (as seen above)