University of Pittsburgh
CS 2001 Module: Bayesian belief networks
Professor Milos Hauskrecht September 12, 2002

Problem assignment

Due: Thursday, September 19, 2002 at noon

Problem 1

Select a problem in which relations between entities (events or facts) are uncertain and build a Bayesian belief network graph for it. The problem should have at least 6 discrete-valued random variables and there should be some apparent independence structure between variables; that is, your resulting graph should not be fully connected. Example problems can be the diagnosis of a technical device such as a printer, or a software system, weather forecast, etc. Try to be inventive.

Part a. Give:

- a description of the semantics of random variables and their values;
- a directed acyclic graph representing the relations between variables.

Part b. Answer the following questions:

- How many parameters define the full joint distribution?
- How many parameters are needed to parameterize your Bayesian belief network?

Note: The most intutitive approach to build a network is to use directed links to model direct causal relations between entities.

Problem 2

Random variables A,B are said to be conditionally independent given C when:

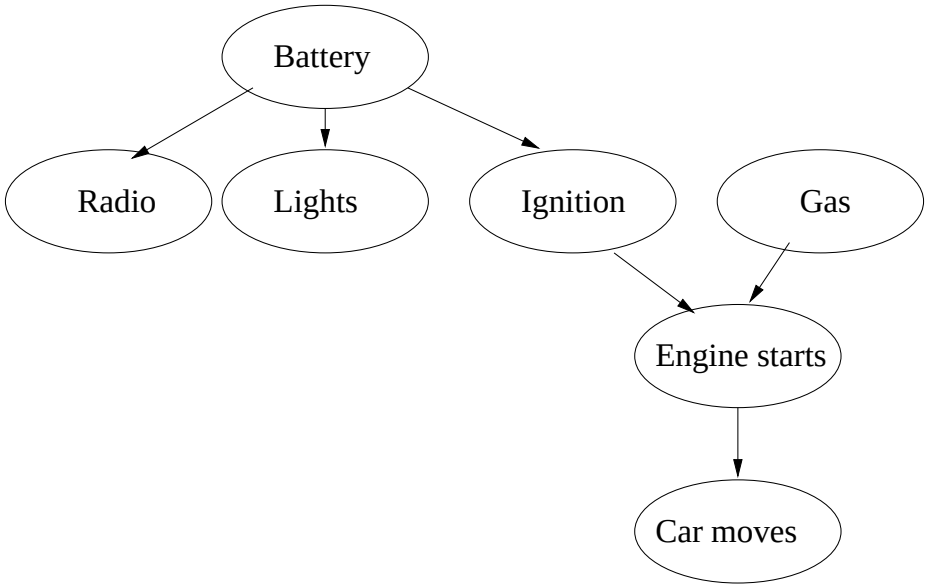
$$P(A, B|C) = P(A|C)P(B|C).$$

Prove that this implies:

$$P(A|B, C) = P(A|C).$$

Problem 3

Assume the Bayesian belief network in the figure below. Assume that every variable in the network is binary and it can take two possible values: True (T) and False (F).



The belief network encodes the full joint distribution over random variables (represented by nodes) by exploiting conditional independences that hold among variables.

Part a. Show how to compute the joint probability P(C=F, E=T, G=T,I=T, L=F,R=T,B=T) using the belief network model. The names of the variables in the network were replaced with their first letters, e.g., Radio with R, or Engine starts with E.

Part b. Assume we want to compute the probability of Car not moving, that is $P(C = F)$. Write down the expression for computing the probability from conditionals via blind approach. What is the inference cost? The cost should be expressed in terms of the number of sums and number of products used.

Part c. Propose a more efficient solution for computing $P(C = F)$ by interleaving sums and products. Write down the new expression and compute the inference cost.