University of Pittsburgh
CS 1571 Introduction to Artificial Intelligence
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Handout 9
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Solutions to problem set 5

Propositional logic

Problem 1. Propositional logic.

Let KB consists of the following sentences:

$$\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T),$$
$$\neg(T \vee Q),$$
$$U \rightarrow (\neg T \rightarrow (\neg S \wedge P)).$$

Prove that $\neg U$ holds using three approaches.

Part a. Truth-table approach.

Answer. The complete truth table for the problem is:

U	P	Q	S	T	$(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$	$\neg(T \vee Q)$	$U- > (\neg T- > (\neg S \wedge P))$	KB	KB true mark
1	1	1	1	1	1	0	0	0	
1	1	1	1	0	1	0	0	0	
1	1	1	0	1	1	0	1	0	
1	1	1	0	0	0	0	1	0	
1	1	0	1	1	1	0	0	0	
1	1	0	1	0	1	1	0	0	
1	1	0	0	1	1	0	1	0	
1	1	0	0	0	1	1	1	0	
1	1	1	1	1	1	0	0	0	
1	0	1	1	0	1	0	0	0	
1	0	1	0	1	1	0	0	0	
1	0	1	0	0	0	0	0	0	
1	0	0	1	1	1	0	0	0	
1	0	0	1	0	1	1	0	0	
1	0	0	0	1	1	0	0	0	
1	0	0	0	0	0	1	0	0	
0	1	1	1	1	1	0	1	0	
0	1	1	1	0	1	0	1	0	
0	1	1	0	1	1	0	1	0	
0	1	1	0	0	0	0	1	0	
0	1	0	1	1	1	0	1	0	
0	1	0	1	0	1	1	1	1	*
0	1	0	0	1	1	0	1	0	
0	1	0	0	0	1	1	1	1	*
0	1	1	1	1	1	0	1	0	
0	0	1	1	0	1	0	1	0	
0	0	1	0	1	1	0	1	0	
0	0	1	0	0	0	0	1	0	
0	0	0	1	1	1	0	1	0	
0	0	0	1	0	1	1	1	1	*
0	0	0	0	1	1	0	1	0	
0	0	0	0	0	0	1	1	0	

The table may be interpreted in two interesting ways.

a) Assignment of True to U makes KB False - proof by refutation.

b) Everywhere KB is true, $\neg U$ is also true - $KB \models \neg U$

Part b. Inference rule approach.

Answer. These are the axioms:

1. $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$
2. $\neg(T \vee Q)$
3. $U -> (\neg T -> (\neg S \wedge P))$

This is a possible inference sequence:

4. $\neg P \vee Q \vee S \vee T$, DeMorgan on 1
5. $\neg U \vee T \vee (\neg S \wedge P)$, from 3 and definition of $\rightarrow$
6. $(\neg U \vee T \vee \neg S) \wedge (\neg U \vee T \vee P)$, from 5 and distribution over $\wedge$
7. $\neg U \vee T \vee \neg S$, and-elimination on 6
8. $\neg U \vee T \vee P$, and-elimination on 6
9. $\neg T \wedge \neg Q$, DeMorgan on 2
10. $\neg T$, and-elimination on 9
11. $\neg Q$, and-elimination on 9
12. $\neg P \vee S \vee T$, unit resolution 4 and 11
13. $\neg P \vee S$, unit resolution 12 and 10
14. $\neg U \vee P$, unit resolution 8 and 10
15. $\neg U \vee \neg S$, unit resolution 7 and 10
16. $\neg U \vee S$, resolution 13, 14
17. $\neg U \vee \neg U$, resolution 15,16
18. $\neg U$, tautology 17 (**the theorem is proved !!**)

Problem 2. Propositional logic.

Assume the following set of facts:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

- **Part a.** Express the above knowledge in the propositional logic.
- **Part b.** Can you prove that the unicorn is mythical? Give a proof if provable.
- **Part c.** Can you prove the fact that the unicorn is magical? Give a proof if provable.
- **Part d.** Can you prove the fact that the unicorn is horned? Give a proof if provable.

Solutions.

Part a. Translation (All sentences like "Unicorn is a mammal" should be understood to be atomical propositional symbols).

1. Unicorn is mythical $\Leftrightarrow$ Unicorn is immortal;
2. $\neg$ Unicorn is mythical $\Rightarrow$ Unicorn is a mammal;
3. Unicorn is immortal $\vee$ Unicorn is a mammal $\Rightarrow$ Unicorn is horned;
4. Unicorn is horned $\Rightarrow$ Unicorn is magical.

Parts b - d. The easiest way to prove all theorems is to use the truth table approach with propositional symbols corresponding to facts: `Unicorn is mythical`, `Unicorn is immortal`, `Unicorn is a mammal`, `Unicorn is horned`, `Unicorn is magical`. However, one can also reason out the results:
From the first two statements (1,2) we see that if `Unicorn is mythical` then `Unicorn is immortal`, otherwise `Unicorn is a mammal`. But then the premise of (3) is always True and thus `Unicorn is horned` is also always TRUE (part d). This causes the premise of (4) to be always TRUE and therefore `Unicorn is magical` must be TRUE (part c). However, we cannot say anything about whether it is mythical, since both `Unicorn is mythical` and $\neg$ `Unicorn is mythical` are consistent with our translations. Thus it cannot be proved to be true.