University of Pittsburgh
CS 1571 Introduction to Artificial Intelligence
Professor Milos Hauskrecht

Handout 1
September 17, 2009

Solutions to problem assignment 1

Problem 1

Assume we want to solve the **map coloring problem**. The goal is to color a map such that no countries on the map that share a border are assigned the same color. The number of colors is limited. For the purpose of assignment assume you have a map with 10 countries and you have three different colors: Green, Red and Blue.

Part a. Formulate the map coloring problem as a (graph) search problem by defining its initial state, operators and the goal condition.

Part b. What is the search space size of your formulation? If the exact calculation of the search space size of your formulation becomes hard, give a reasonable upper bound estimate.

Solutions.

Part a

1. **States:** A colorless, partially-colored or fully colored map, with the countries.
2. **Initial state:** The colorless map, with the countries
3. **Operator:** Select an adjecent to a colored country, and appy any one of the color, such that there is no conflict of colors. (choose the first country to be colored at random)
4. **Goal State:** All the countries are colored and no tow adjecent countries have the same color

Part b

Suppose there are n number of countries, and r number of colors (in this particular problem, n=10 and r= 3) . At the first stage, any one country is picked, and colored with any one of the r colors. Thus there are r possibilities at this stage. In the next level, we pick the adjecent country, and there are r possibilities ar this stage, for *every one* of the color selection in the previous stage. Thus there are r^2 possibilities at this level. As there are n countries to be colored, after n levels in the search tree, there will be no more countries to be colored. At this last level, there will be r^n possibilities.

Thus the search space can be said to be of the order of $O(r^n)$

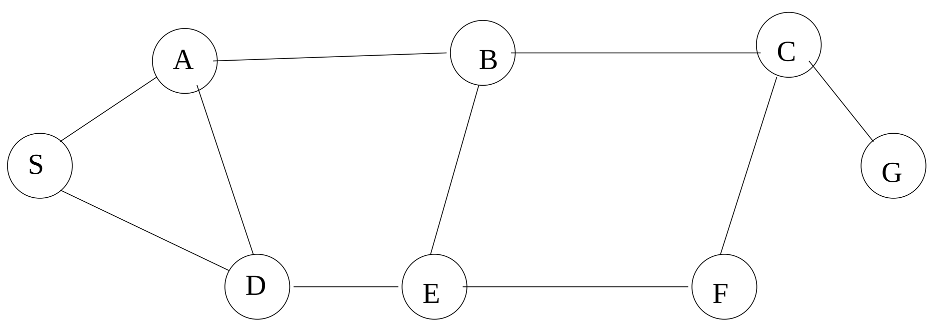
To be more precise, the exact search speace will be

$$\sum_{i=1}^{n} r^i$$

We can also get tighter bounds than this by considering the fact that once one country at a level is colored with a particular color,then its adjecent country cannot be given that color. Hence it would have to choose from (r-1) colors only.

Problem 2

Consider the following graph representing road connections between different cities. Let S be the initial city and G the destination.



Part a. Show how the depth-first search (DFS) would search the graph. That is, give an order (of first 10 nodes) in which the nodes could be *expanded*.

Part b. Is the depth first search (DFS) threatened by cycles? That is, is it possible that the DFS program can become stuck in the cycle and hence miss the solution?

Part c. Show how the breadth-first-search would search the graph.

Part d. Can the BFS program be ever become stuck in the cycle and miss the solution?

Solutions. The search tree in the figure below illustrates two search algorithms of part a. c. The figures show the search tree built by different algorithms before the goal (node G) was reached. Crossed nodes were eliminated using the corresponding criterion applied and were never expanded. Dashed lines reflect blind-alley branches that were searched during the depth-first search.

Part a.

A possible order: S,A,B,C,G.

Part b.

Yes, It will be stuck in the cycle. For example, S, A, S

Part c.

A possible order: S,A,D,B,E,C,F,G.

Part d.

No. If the solution exists we will find it and possible cycles won't prevent us from reaching.