CS 1571 Introduction to AI
Lecture 21

Decision making in the presence of uncertainty

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Decision-making in the presence of uncertainty

- Computing the probability of some event may not be our ultimate goal
- Instead we are often interested in **making decisions about our future actions so that we satisfy some goals**
- **Example: medicine**
  – Diagnosis is typically only the first step
  – The ultimate goal is to manage the patient in the best possible way. Typically many options available:
    - Surgery, medication, collect the new info (lab test)
    - There is an **uncertainty in the outcomes** of these procedures: patient can be improve, get worse or even die as a result of different management choices.

Decision-making in the presence of uncertainty

Main issues:

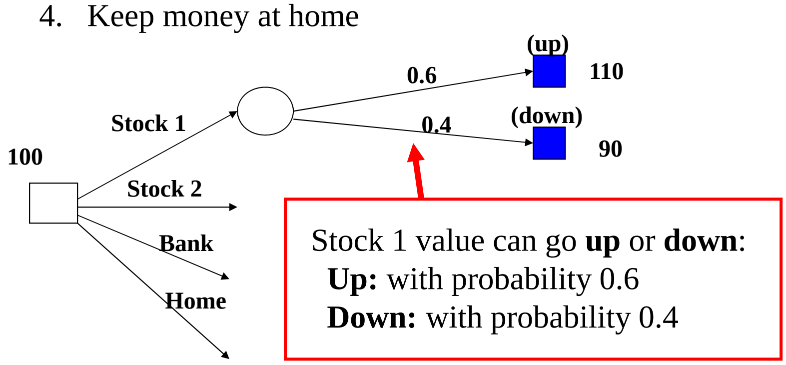
- **How to model the decision process with uncertain outcomes in the computer ?**
- **How to make decisions about actions in the presence of uncertainty?**

The field of **decision-making** studies ways of making decisions in the presence of uncertainty.

Decision making example.

Assume we want to invest $100 for 6 months

- **We have 4 choices:**
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home



Decision making example.

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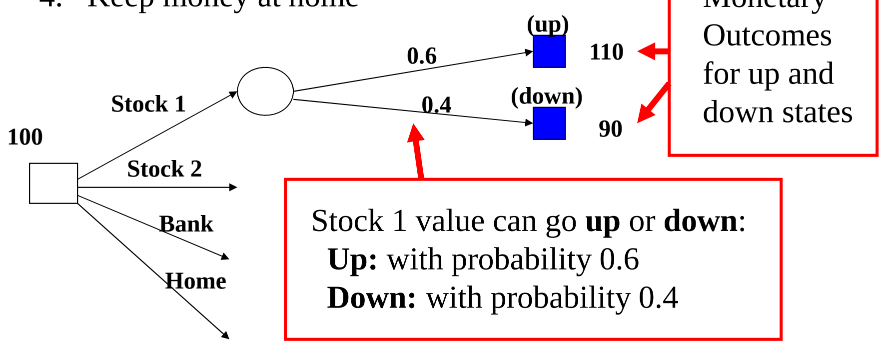
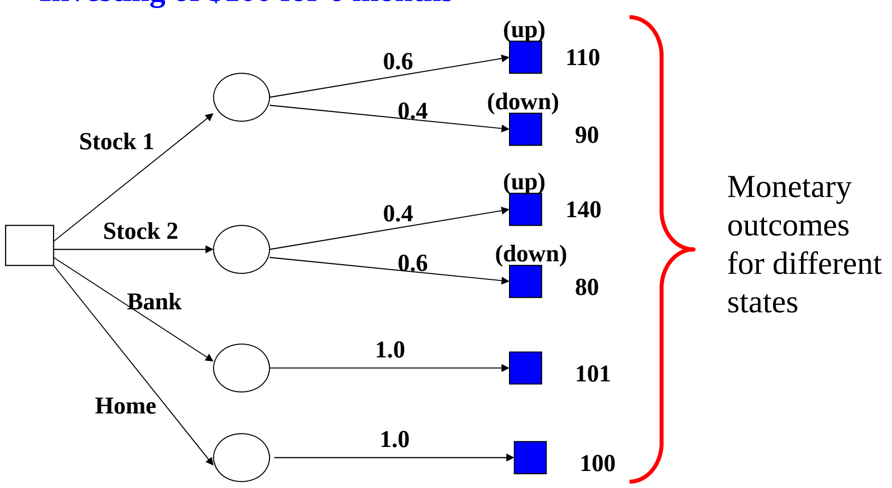


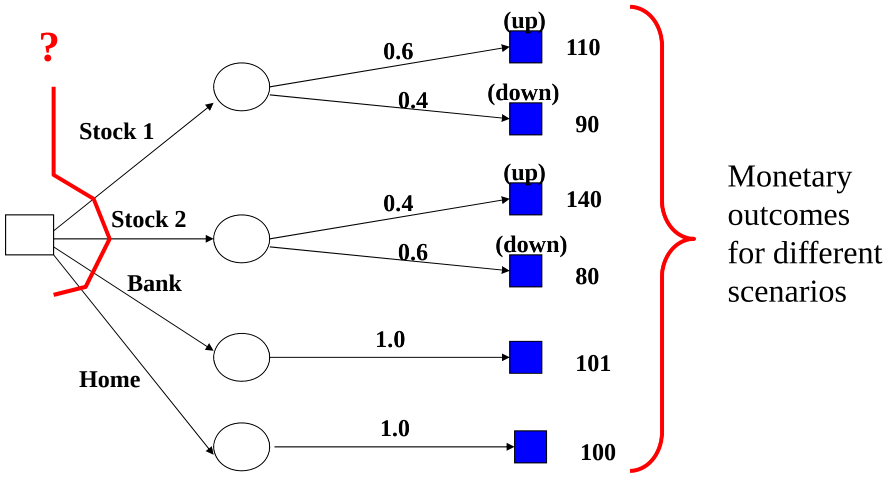
Decision making example.

Investing of $100 for 6 months



Decision making example.

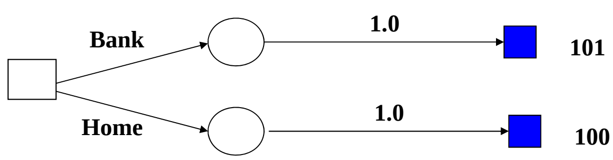
We need to make a choice whether to invest in Stock 1 or 2, put money into bank or keep them at home. But how?



Decision making example.

Assume a simplified problem with the Bank and Home choices only.

The result is guaranteed – the outcome is deterministic

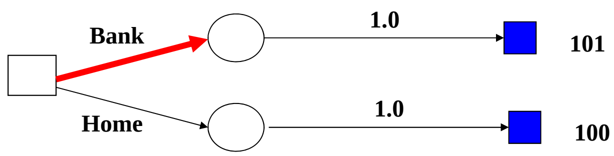


What is the rational choice assuming our goal is to make money?

Decision making. Deterministic outcome.

Assume a simplified problem with the Bank and Home choices only.

These choices are deterministic.



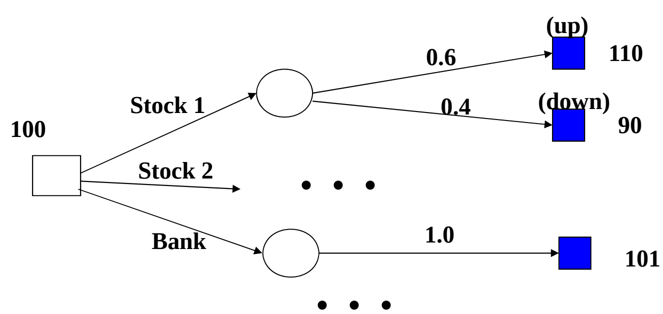
Our goal is to make money. What is the rational choice?

Answer: Put money into the bank. The choice is always strictly better in terms of the outcome

But what to do if we have uncertain outcomes?

Decision making. Stochastic outcome

- **How to quantify the goodness of the stochastic outcome?**
  We want to compare it to deterministic and other stochastic outcomes.



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Decision making. Stochastic outcome

- **How to quantify the goodness of the stochastic outcome?**
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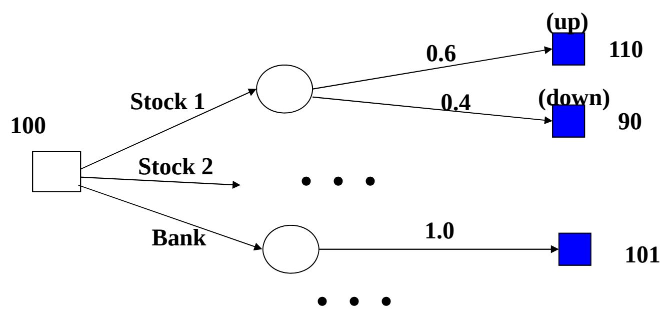


Idea: **Use the expected value of the outcome**

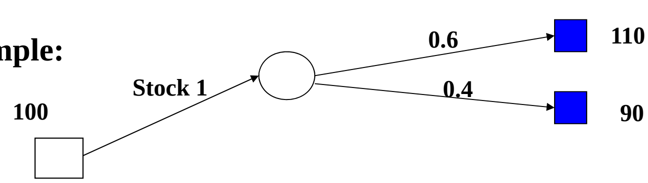
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X .
- **Expected value** of X is:

$$E(X) = \sum_{x \in \Omega_X} xP(X = x)$$

Intuition: Expected value summarizes all stochastic outcomes into a single quantity.

- **Example:**



- What is the expected value of the outcome of Stock 1 option?

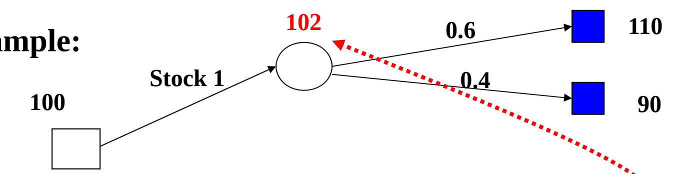
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X .
- **Expected value** of X is:

$$E(X) = \sum_{x \in \Omega_X} x P(X = x)$$

  - **Expected value** summarizes all stochastic outcomes into a single quantity

- **Example:**

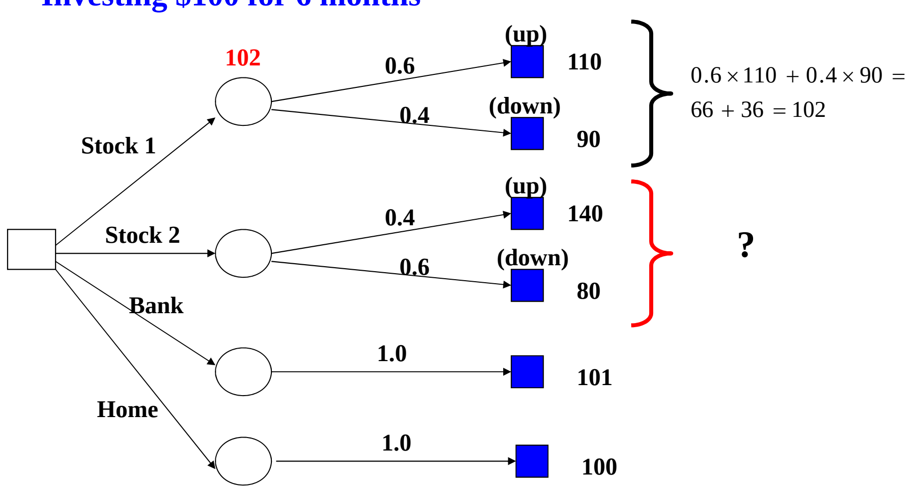


Expected value for the outcome of the Stock 1 option is:

$$0.6 \times 110 + 0.4 \times 90 = 66 + 36 = \boxed{102}$$

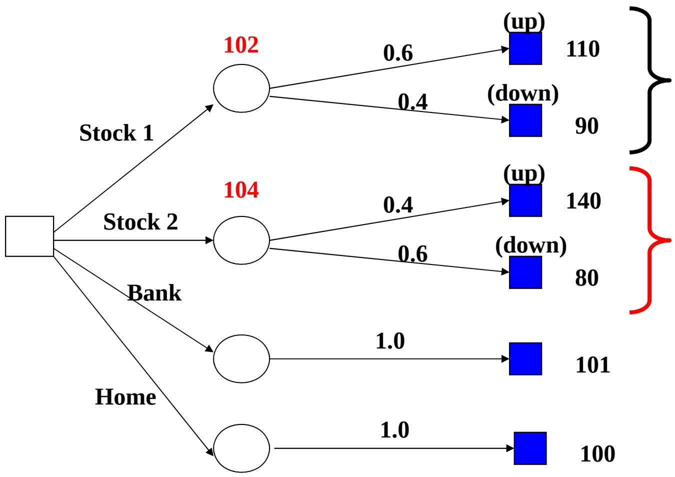
Expected values

Investing \$100 for 6 months



Expected values

Investing $100 for 6 months

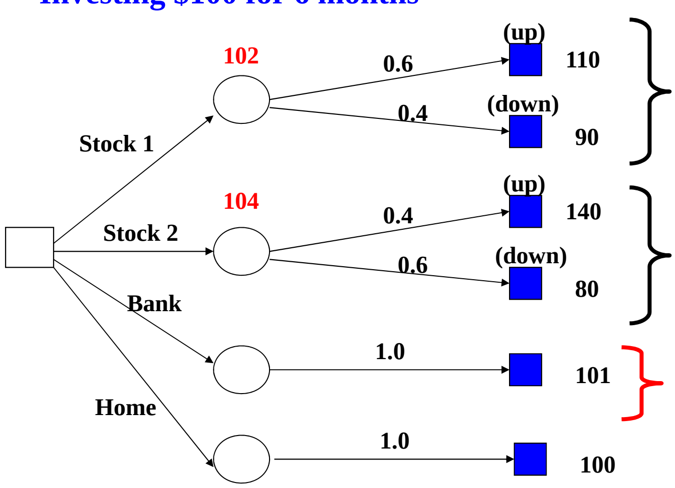


$0.6 \times 110 + 0.4 \times 90 =$
$66 + 36 = 102$

$0.4 \times 140 + 0.6 \times 80 =$
$56 + 48 = 104$

Expected values

Investing $100 for 6 months



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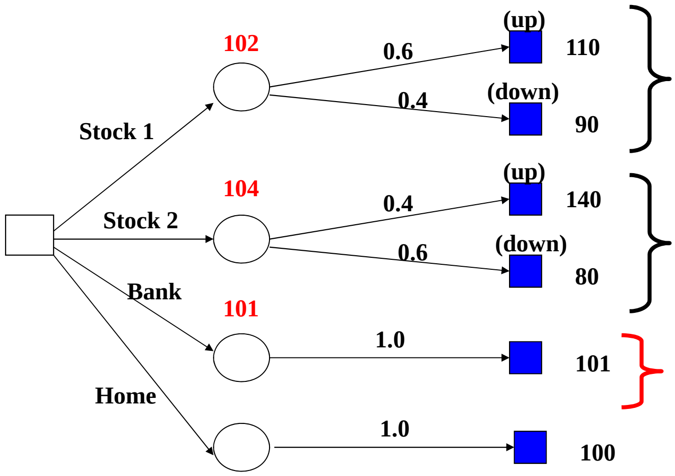
Expected values

Investing $100 for 6 months



$0.6 \times 110 + 0.4 \times 90 =$
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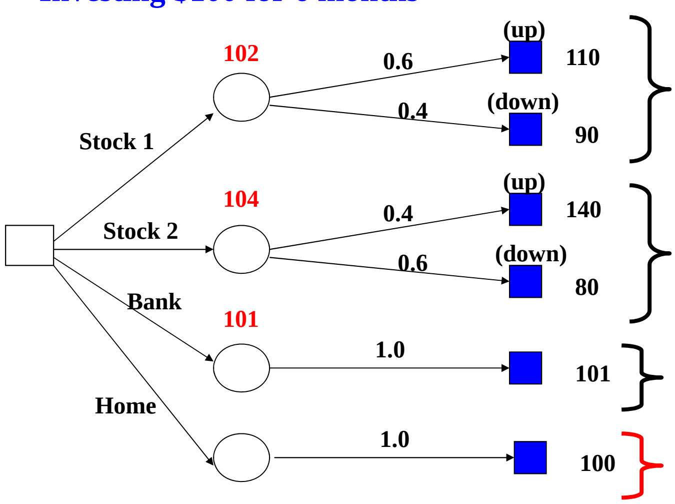
$1.0 \times 101 = 101$

Expected values

Investing $100 for 6 months



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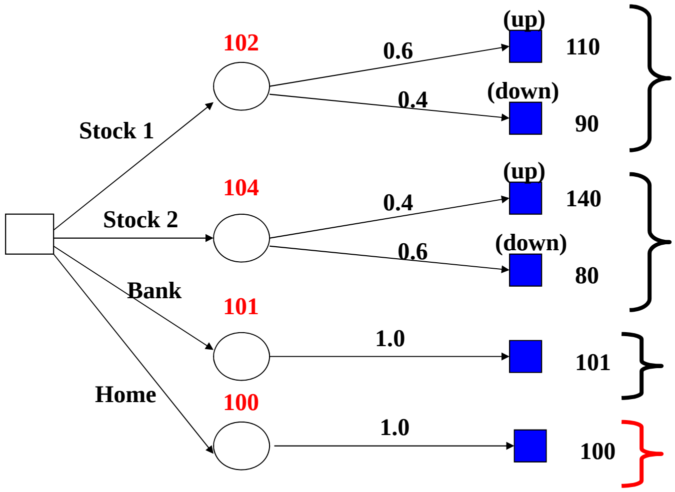
Expected values

Investing $100 for 6 months



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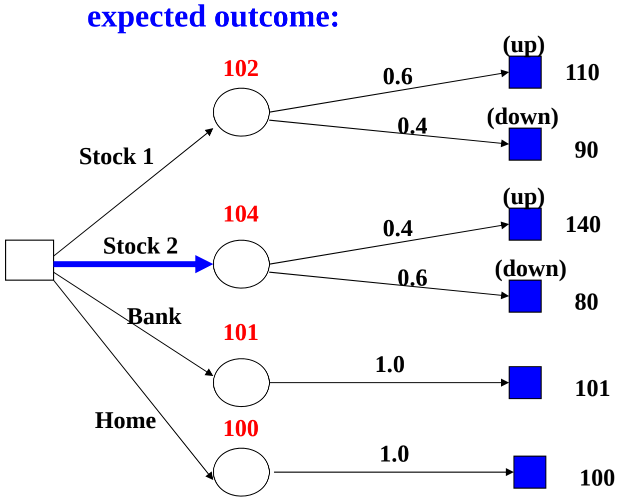
$1.0 \times 100 = 100$

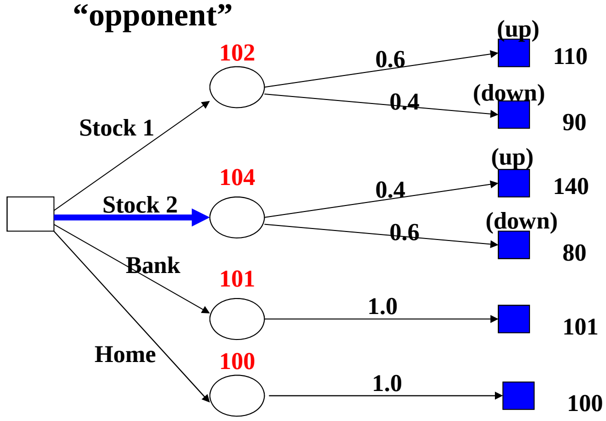
Selection based on expected values

The optimal action is the option that maximizes the expected outcome:



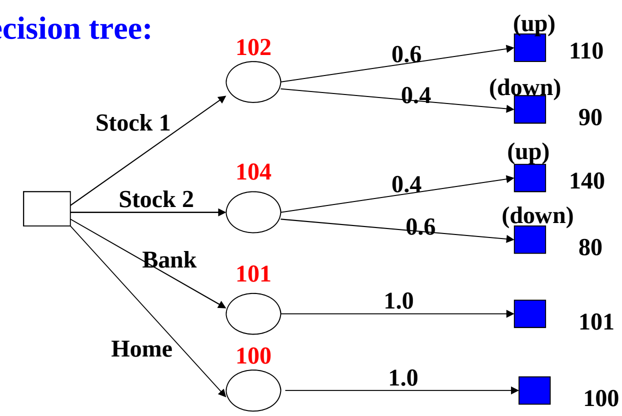
Relation to the game search

- **Game search: minimax algorithm**
  - **considers the rational opponent and its best move**
- **Decision making: maximizes the expectation**
  - **play against the nature – a stochastic non-malicious "opponent"**



(Stochastic) Decision tree

- **Decision tree:**



- □ **decision node**
- ◯ **chance node**
- ■ **outcome (value) node**

Sequential (multi-step) problems

The decision tree can be build to capture multi-step decision problems:

- Choose an action
- Observe the stochastic outcome
- And repeat

How to make decisions for multi-step problems?

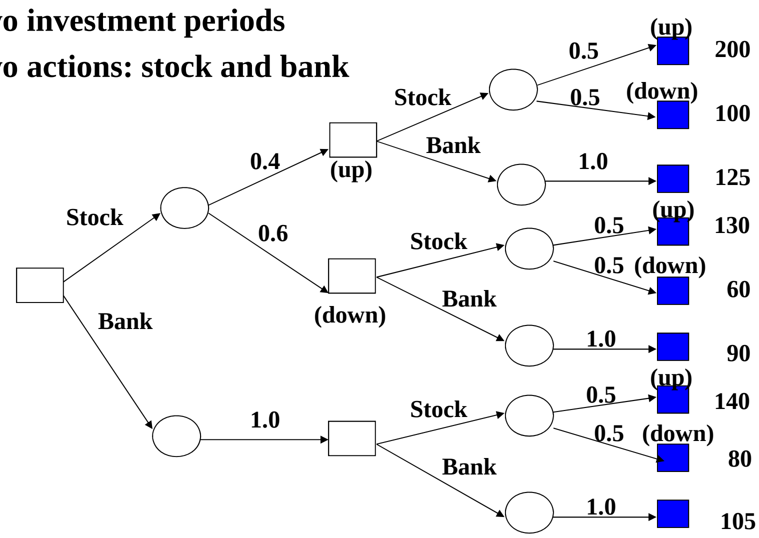
- Start from the leaves of the decision tree (outcome nodes)
- Compute expectations at chance nodes
- Maximize at the decision nodes

Algorithm is sometimes called **expectimax**

Multi-step problem example

Assume:

- **Two investment periods**
- **Two actions: stock and bank**



Multi-step problem example

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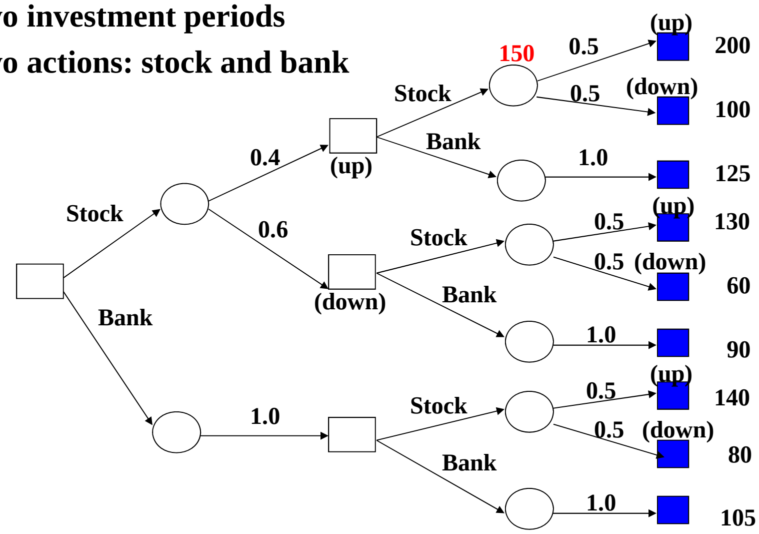

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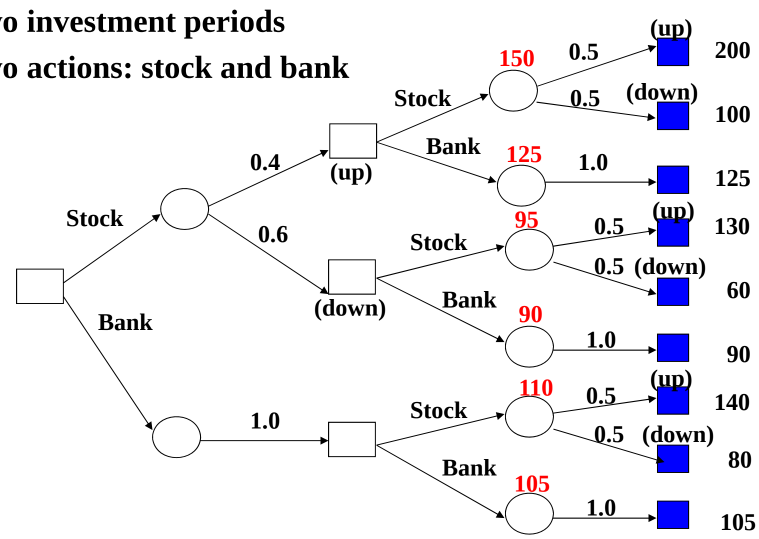

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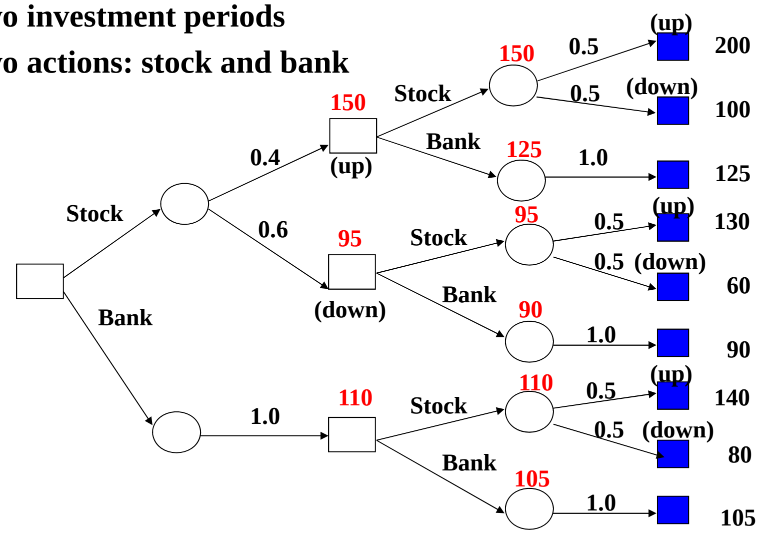

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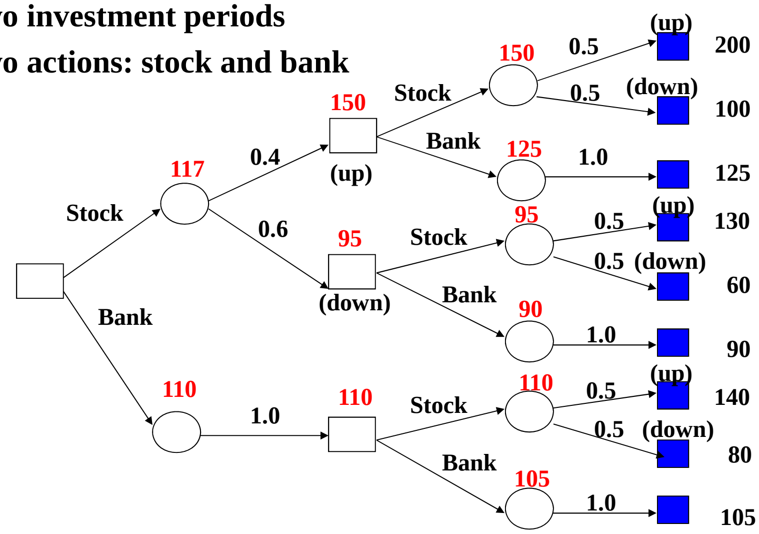

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