

Sample Problems (with answers) from Fall 2006 Final

1. **Chapter 5: Counting**

- (a) **(4 pts)** Imagine that each locker in Peterson is labeled with an upper-case letter followed by three digits.
- How many different labels for lockers are there?  $26 \cdot 10 \cdot 10 \cdot 10$
  - Name the rule(s) that you used to compute the number of labels.  
Product
- (b) **(4 pts)** Now imagine that there are 805 lockers in Peterson and 4026 students who need lockers. Therefore, some students must share lockers.
- What is the largest number of students who must necessarily share a locker? 6
  - Name the rule(s) that you used to compute the number of students.  
Pigeonhole
- (c) **(5 pts)** Imagine a license plate consists of three letters followed by three digits or four letters followed by two digits.
- How many different license plates can be made?  $26^3 \cdot 10^3 + 26^4 \cdot 10^2$  (\*\* means exponentiation, \* means multiplication)
  - Name the rule(s) that you used to compute the number of license plates. Product and Sum
- (d) **(4 pts)** Let the set  $A = \{1,2,3\}$ .
- List all of the 2-permutations of the set A. The 6 ordered arrangements 1,2 ; 1,3 ; 2,1; 2,3; 3,1 ; 3,2
  - List all of the 2-combinations of the set A. The 3 subsets  $\{1,2\}$ ,  $\{1,3\}$ ,  $\{2,3\}$
  - Verify the size of your list by using the formulas for permutations and combinations.  $P(3,2) = 3 \cdot 2 = 6$ ;  $C(3,2) = 3! / (2!1!) = 3$

## 2. Chapter 6: Discrete Probability

- (a) **(2 pts)** What is the probability that a fair coin lands heads four times out of five flips? Reduce to a fraction.  $C(5,4)/(2^{**}5) = 5/32$
- (b) **(3 pts)** What is the probability that a positive integer less than 100 picked at random has all distinct digits?  $90/99$
- (c) **(6 pts)** A computer picks out at random a sequence of six digits.
- What is the probability that a person picks all six digits in their correct positions?  $1/(10^{**}6)$
  - What is the probability that a person picks exactly five of the digits, in the correct order?  $54/(10^{**}6)$