

Problem from Section 2.4

16. a) The terms of this sequence alternate between 2 (if j is even) and 0 (if j is odd). Thus the sum is $2 + 0 + 2 + 0 + 2 + 0 + 2 + 0 + 2 = 10$.

b) We can break this into two parts and compute $(\sum_{j=0}^8 3^j) - (\sum_{j=0}^8 2^j)$. Each summation can be computed from the formula for the sum of a geometric progression. Thus the answer is

$$\frac{3^9 - 1}{3 - 1} - \frac{2^9 - 1}{2 - 1} = 9841 - 511 = 9330.$$

c) As in part (b) we can break this into two parts and compute $(\sum_{j=0}^8 2 \cdot 3^j) + (\sum_{j=0}^8 3 \cdot 2^j)$. Each summation can be computed from the formula for the sum of a geometric progression. Thus the answer is

$$\frac{2 \cdot 3^9 - 2}{3 - 1} + \frac{3 \cdot 2^9 - 3}{2 - 1} = 19682 + 1533 = 21215.$$

d) This could be worked as in part (b), but it is easier to note that the sum telescopes (see Exercise 19). Each power of 2 cancels except for the -2^0 when $j = 0$ and the 2^9 when $j = 8$. Therefore the answer is $2^9 - 2^0 = 511$. (Alternatively, note that $2^{j+1} - 2^j = 2^j$.)

18. We will just write out the sums explicitly in each case.

a) $(1 - 1) + (1 - 2) + (2 - 1) + (2 - 2) + (3 - 1) + (3 - 2) = 3$

b) $(0 + 0) + (0 + 2) + (0 + 4) + (3 + 0) + (3 + 2) + (3 + 4) + (6 + 0) + (6 + 2) + (6 + 4) + (9 + 0) + (9 + 2) + (9 + 4) = 7$

c) $(0 + 1 + 2) + (0 + 1 + 2) + (0 + 1 + 2) = 9$

d) $(0 + 0 + 0 + 0) + (0 + 1 + 8 + 27) + (0 + 4 + 32 + 108) = 180$