CS441 – Discrete Structures for Computer Science
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Problem from Section 2.4

4. a) $a_0 = (-2)^0 = 1$, $a_1 = (-2)^1 = -2$, $a_2 = (-2)^2 = 4$, $a_3 = (-2)^3 = -8$
   b) $a_0 = a_1 = a_2 = a_3 = 3$
   c) $a_0 = 7 + 4^0 = 8$, $a_1 = 7 + 4^1 = 11$, $a_2 = 7 + 4^2 = 23$, $a_3 = 7 + 4^3 = 71$
   d) $a_0 = 2^0 + (-2)^0 = 2$, $a_1 = 2^1 + (-2)^1 = 0$, $a_2 = 2^2 + (-2)^2 = 8$, $a_3 = 2^3 + (-2)^3 = 0$

6. These are easy to compute by hand, calculator, or computer.
   a) 10, 7, 4, 1, −2, −5, −8, −11, −14, −17
   b) We can use the formula in Table 2, or we can just keep adding to the previous term ($1 + 2 = 3$, $3 + 3 = 6$, $6 + 4 = 10$, and so on): 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. These are called the triangular numbers.
   c) 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025
   d) 1, 1, 1, 2, 2, 2, 2, 2, 3, 3 (there will be $2k + 1$ copies of k) e) 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
   f) The largest number whose binary expansion has n bits is $(11\ldots1)_2$, which is $2^n - 1$. So the sequence is 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023.
   g) 1, 2, 2, 4, 8, 11, 33, 37, 148, 153 h) 1, 2, 2, 2, 2, 3, 3, 3, 3, 3

10. a) The first term is 3, and the n^{th} term is obtained by adding $2n - 1$ to the previous term. In other words, we successively add 3, then 5, then 7, and so on. Alternatively, we see that the n^{th} term is $n^2 + 2$; we can see this by inspection if we happen to notice how close each term is to a perfect square, or we can fit a quadratic polynomial to the data. The next three terms are 123, 146, 171.
   b) This is an arithmetic sequence whose first term is 7 and whose difference is 4. Thus the n^{th} term is $7 + 4(n - 1) = 4n + 3$. Thus the next three terms are 47, 51, 55.
   c) The n^{th} term is clearly the binary expansion of n. Thus the next three terms are 1100, 1101, 1110.
   d) The sequence consists of one 1, followed by three 2's, followed by five 3's, followed by seven 5's, and so on, with the number of copies of the next value increasing by 2 each time, and the values themselves following the rule that the first two values are 1 and 2 and each subsequent value is the sum of the previous two values. Obviously other answers are possible as well. By our rule, the next three terms would be 8, 8, 8.
   e) If we stare at this sequence long enough and compare it with Table 1, then we notice that the n^{th} term is $3^n - 1$. Thus the next three terms are 59048, 177146, 531440.
   f) We notice that each term evenly divides the next, and the multipliers are successively 3, 5, 7, 9, 11, and so on. That must be the intended pattern. One notation for this is to use $n!!$ to mean $n(n-2)(n-4)\cdots$; thus the n^{th} term is $(2n-1)!!$. Thus the next three terms are 654729075, 13749310575, 316234143225.
   g) The sequence consists of one 1, followed by two 0s, then three 1s, four 0s, five 1s, and so on, alternating between 0s and 1s and having one more item in each group than in the previous group. Thus six 0's will follow next, so the next three terms are 0, 0, 0.
   h) It doesn't take long to notice that each term is the square of its predecessor. The next three terms get very big very fast: 18446744073709551616, 340282366920938463463374607431768211456, and then

$$115792089237316195423570985008687907853269984665640564039457584007913129639936.$$

(These were computed using *Maple*.)