Problem from Section 8.1

d) Since $1 \neq 2 \cdot 1$, this relation is not reflexive. It is not symmetric, since $(2,1) \in R$, but $(1,2) \notin R$. To see that it is antisymmetric, suppose that $x = 2y$ and $y = 2x$. Then $y = 4y$, from which it follows that $y = 0$ and hence $x = 0$. Thus the only time that (x,y) and (y,x) are both is R is when $x = y$ (and both are 0). This relation is clearly not transitive, since $(4,2) \in R$ and $(2,1) \in R$, but $(4,1) \notin R$.

e) This relation is reflexive since squares are always nonnegative. It is clearly symmetric (the roles of x and y in the statement are interchangeable). It is not antisymmetric, since $(2,3)$ and $(3,2)$ are both in R. It is not transitive; for example, $(1,0) \in R$ and $(0,-2) \in R$, but $(1,-2) \notin R$.

f) This is not reflexive, since $(1,1) \notin R$. It is clearly symmetric (the roles of x and y in the statement are interchangeable). It is not antisymmetric, since $(2,0)$ and $(0,2)$ are both in R. It is not transitive; for example, $(1,0) \in R$ and $(0,-2) \in R$, but $(1,-2) \notin R$.

g) This is not reflexive, since $(2,2) \notin R$. It is not symmetric, since $(1,2) \in R$ but $(2,1) \notin R$. It is antisymmetric, because if $(x,y) \in R$ and $(y,x) \in R$, then $x = 1$ and $y = 1$, so $x = y$. It is transitive, because if $(x,y) \in R$ and $(y,z) \in R$, then $x = 1$ (and $y = 1$, although that doesn't matter), so $(x,z) \in R$.

h) This is not reflexive, since $(2,2) \notin R$. It is clearly symmetric (the roles of x and y in the statement are interchangeable). It is not antisymmetric, since $(2,1)$ and $(1,2)$ are both in R. It is not transitive; for example, $(3,1) \in R$ and $(1,7) \in R$, but $(3,7) \notin R$.

30. Since $(1,2) \in R$ and $(2,1) \in S$, we have $(1,1) \in S \circ R$. We use similar reasoning to form the rest of the pairs in the composition, giving us the answer $\{(1,1),(1,2),(2,1),(2,2)\}$.

SECTION 8.5

2. a) This is an equivalence relation by Exercise 9 ($f(x)$ is x's age).
   b) This is an equivalence relation by Exercise 9 ($f(x)$ is x's parents).
   c) This is not an equivalence relation, since it need not be transitive. (We assume that biological parentage is at issue here, so it is possible for A to be the child of W and X, B to be the child of X and Y, and C to be the child of Y and Z. Then A is related to B, and B is related to C, but A is not related to C.)
   d) This is not an equivalence relation since it is clearly not transitive.
   e) Again, just as in part (c), this is not transitive.

8. Recall (Definition 4 in Section 2.4) that two sets have the same cardinality if there is a bijection (one-to-one and onto function) from one set to the other. We must show that R is reflexive, symmetric, and transitive. Every set has the same cardinality as itself because of the identity function. If f is a bijection from S to T, then f^{-1} is a bijection from T to S, so R is symmetric. Finally, if f is a bijection from S to T and g is a bijection from T to U, then $g \circ f$ is a bijection from T to U, so R is transitive (see Exercise 29 in Section 2.3).

12. This follows from Exercise 9, where f is the function that takes a bit string of length $n \geq 3$ to its last $n-3$ bits.