Pseudo Homework Solutions — *University of Pittsburgh – Computer Science Department*

CS441 – Discrete Structures for Computer Science
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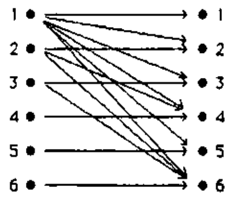
Problem from Section 8.1

2. a) (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)

b) We draw a line from a to b whenever a divides b, using separate sets of points; an alternate form of this graph would have just one set of points.



c) We put an $\times$ in the i^{th} row and j^{th} column if and only if i divides j.

R	1	2	3	4	5	6
1	×	×	×	×	×	×
2		×		×		×
3			×			×
4				×		
5					×	
6						×

4. a) Being taller than is not reflexive (I am not taller than myself), nor symmetric (I am taller than my daughter, but she is not taller than I). It is antisymmetric (vacuously, since we never have A taller than B, and B taller than A, even if $A = B$). It is clearly transitive.

b) This is clearly reflexive, symmetric, and transitive (it is an equivalence relation—see Section 8.5). It is not antisymmetric, since twins, for example, are unequal people born on the same day.

c) This has exactly the same answers as part (b), since having the same first name is just like having the same birthday.

d) This is clearly reflexive and symmetric. It is not antisymmetric, since my cousin and I have a common grandparent, and I and my cousin have a common grandparent, but I am not equal to my cousin. This relation is not transitive. My cousin and I have a common grandparent; my cousin and her cousin on the other side of her family have a common grandparent. My cousin's cousin and I do not have a common grandparent.

6. a) Since $1 + 1 \neq 0$, this relation is not reflexive. Since $x + y = y + x$, it follows that $x + y = 0$ if and only if $y + x = 0$, so the relation is symmetric. Since $(1, -1)$ and $(-1, 1)$ are both in R, the relation is not antisymmetric. The relation is not transitive: for example, $(1, -1) \in R$ and $(-1, 1) \in R$, but $(1, 1) \notin R$.

b) Since $x = \pm x$ (choosing the plus sign), the relation is reflexive. Since $x = \pm y$ if and only if $y = \pm x$, the relation is symmetric. Since $(1, -1)$ and $(-1, 1)$ are both in R, the relation is not antisymmetric. The relation is transitive, essentially because the product of 1's and -1's is ± 1.

c) The relation is reflexive, since $x - x = 0$ is a rational number. The relation is symmetric, because if $x - y$ is rational, then so is $-(x - y) = y - x$. Since $(1, -1)$ and $(-1, 1)$ are both in R, the relation is not antisymmetric. To see that the relation is transitive, not that if $(x, y) \in R$ and $(y, z) \in R$, then $x - y$ and $y - z$ are rational numbers. Therefore their sum $x - z$ is rational, and that means that $(x, z) \in R$.