CS441 – Discrete Structures for Computer Science
Instructor: Dr.Litman

Problem from Section 5.2

2. This follows from the pigeonhole principle, with $k = 26$.

4. We assume that the woman does not replace the balls after drawing them.

a) There are two colors: these are the pigeonholes. We want to know the least number of pigeons needed to insure that at least one of the pigeonholes contains three pigeons. By the generalized pigeonhole principle, the answer is 5. If five balls are selected, at least $\lceil 5/2 \rceil = 3$ must have the same color. On the other hand four balls is not enough, because two might be red and two might be blue. Note that the number of balls was irrelevant (assuming that it was at least 5).

b) She needs to select 13 balls in order to insure at least three blue ones. If she does so, then at most 10 of them are red, so at least three are blue. On the other hand, if she selects 12 or fewer balls, then 10 of them could be red, and she might not get her three blue balls. This time the number of balls did matter.

14. **a)** We can group the first ten positive integers into five subsets of two integers each, each subset adding up to 11: $\{1,10\}$, $\{2,9\}$, $\{3,8\}$, $\{4,7\}$, and $\{5,6\}$. If we select seven integers from this set, then by the pigeonhole principle at least two of them come from the same subset. Furthermore, if we forget about these two in the same group, then there are five more integers and four groups; again the pigeonhole principle guarantees two integers in the same group. This gives us two pairs of integers, each pair from the same group. In each case these two integers have a sum of 11, as desired.

b) No. The set $\{1,2,3,4,5,6\}$ has only 5 and 6 from the same group, so the only pair with sum 11 is 5 and 6.

18. **a)** If not, then there would be 4 or fewer male students and 4 or fewer female students, so there would be $4+4=8$ or fewer students in all, contradicting the assumption that there are 9 students in the class.

b) If not, then there would be 2 or fewer male students and 6 or fewer female students, so there would be $2+6=8$ or fewer students in all, contradicting the assumption that there are 9 students in the class.

40. Look at the pigeonholes $\{1000,1001\}$, $\{1002,1003\}$, $\{1004,1005\}$, $\ldots$, $\{1098,1099\}$. There are clearly 50 sets in this list. By the pigeonhole principle, if we have 51 numbers in the range from 1000 to 1099 inclusive, then at least two of them must come from the same set. These are the desired two consecutive house numbers.