CS441 – Discrete Structures for Computer Science
Instructor: Dr.Litman

Problem from Section 6.3

2. We know that $p(E \mid F) = p(E \cap F)/p(F)$, so we need to find those two quantities. We are given $p(F) = 3/4$. To compute $p(E \cap F)$, we can use the fact that $p(E \cap F) = p(E)p(F \mid E)$. We are given that $p(E) = 2/3$ and that $p(F \mid E) = 5/8$; therefore $p(F \cap E) = (2/3)(5/8) = 5/12$. Putting this together, we have $p(E \mid F) = (5/12)/(3/4) = 5/9$.

4. Let F be the event that Ann picks the second box. Thus we know that $p(F) = p(\overline{F}) = 1/2$. Let B be the event that Frida picks an orange ball. Because of the contents of the boxes, we know that $p(B \mid F) = 5/11$ (five of the eleven balls in the second box are orange) and $p(B \mid \overline{F}) = 3/7$. We are asked for $p(F \mid B)$. We use Bayes' Theorem:

$$p(F \mid B) = \frac{p(B \mid F)p(F)}{p(B \mid F)p(F) + p(B \mid \overline{F})p(\overline{F})} = \frac{(5/11)(1/2)}{(5/11)(1/2) + (3/7)(1/2)} = \frac{35}{68}$$

10. Let A be the event that a randomly chosen person in the clinic is infected with avian influenza. We are told that $p(A) = 0.04$ and therefore $p(\overline{A}) = 0.96$. Let P be the event that a randomly chosen person tests positive for avian influenza on the blood test. We are told that $p(P \mid A) = 0.97$ and $p(P \mid \overline{A}) = 0.02$ ("false positive"). From these we can conclude that $p(\overline{P} \mid A) = 0.03$ ("false negative") and $p(\overline{P} \mid \overline{A}) = 0.98$.

a) We are asked for $p(A \mid P)$. We use Bayes' Theorem:

$$p(A \mid P) = \frac{p(P \mid A)p(A)}{p(P \mid A)p(A) + p(P \mid \overline{A})p(\overline{A})} = \frac{(0.97)(0.04)}{(0.97)(0.04) + (0.02)(0.96)} \approx 0.669$$

b) In part (a) we found $p(A \mid P)$. Here we are asked for the probability of the complementary event (given a positive test result). Therefore we have simply $p(\overline{A} \mid P) = 1 - p(A \mid P) \approx 1 - 0.669 = 0.331$.

c) We are asked for $p(A \mid \overline{P})$. We use Bayes' Theorem:

$$p(A \mid \overline{P}) = \frac{p(\overline{P} \mid A)p(A)}{p(\overline{P} \mid A)p(A) + p(\overline{P} \mid \overline{A})p(\overline{A})} = \frac{(0.03)(0.04)}{(0.03)(0.04) + (0.98)(0.96)} \approx 0.001$$

d) In part (c) we found $p(A \mid \overline{P})$. Here we are asked for the probability of the complementary event (given a negative test result). Therefore we have simply $p(\overline{A} \mid \overline{P}) = 1 - p(A \mid \overline{P}) \approx 1 - 0.001 = 0.999$.

18. We follow the procedure in Example 3. We first compute that $p(\text{exciting}) = 40/500 = 0.08$ and $q(\text{exciting}) = 25/200 = 0.125$. Then we compute that

$$r(\text{exciting}) = \frac{p(\text{exciting})}{p(\text{exciting}) + q(\text{exciting})} = \frac{0.08}{0.08 + 0.125} \approx 0.390.$$

Because $r(\text{exciting})$ is less than the threshold 0.9, an incoming message containing "exciting" would not be rejected.