CS441 – Discrete Structures for Computer Science
Instructor: Dr.Litman

Problem from Section 5.3

12. a) To specify a bit string of length 12 that contains exactly three 1's, we simply need to choose the three positions that contain the 1's. There are $C(12,3) = 220$ ways to do that.

b) To contain at most three 1's means to contain three 1's, two 1's, one 1, or no 1's. Reasoning as in part (a), we see that there are $C(12,3)+C(12,2)+C(12,1)+C(12,0) = 220+66+12+1 = 299$ such strings.

c) To contain at least three 1's means to contain three 1's, four 1's, five 1's, six 1's, seven 1's, eight 1's, nine 1's, 10 1's, 11 1's, or 12 1's. We could reason as in part (b), but we would have too many numbers to add. A simpler approach would be to figure out the number of ways not to have at least three 1's (i.e., to have two 1's, one 1, or no 1's) and then subtract that from 2^{12}, the total number of bit strings of length 12. This way we get $4096 - (66+12+1) = 4017$.

d) To have an equal number of 0's and 1's in this case means to have six 1's. Therefore the answer is $C(12,6) = 924$.

22. a) If ED is to be a substring, then we can think of that block of letters as one superletter, and the problem is to count permutations of seven items—the letters A, B, C, F, G, and H, and the superletter ED. Therefore the answer is $P(7,7) = 7! = 5040$.

b) Reasoning as in part (a), we see that the answer is $P(6,6) = 6! = 720$.

c) As in part (a), we glue BA into one item and glue FGH into one item. Therefore we need to permute five items, and there are $P(5,5) = 5! = 120$ ways to do it.

d) This is similar to part (c). Glue AB into one item, glue DE into one item, and glue GH into one item, producing five items, so the answer is $P(5,5) = 5! = 120$.

e) If both CAB and BED are substrings, then $CABED$ has to be a substring. So we are really just permuting four items: $CABED$, F, G, and H. Therefore the answer is $P(4,4) = 4! = 24$.

f) There are no permutations with both of these substrings, since B cannot be followed by both C and F at the same time.

26. a) This is just a matter of choosing 10 players from the group of 13, since we are not told to worry about what positions they play; therefore the answer is $C(13,10) = 286$.

b) This is the same as part (a), except that we need to worry about the order in which the choices are made, since there are 10 distinct positions to be filled. Therefore the answer is $P(13,10) = 13!/3! = 1{,}037{,}836{,}800$.

c) There is only one way to choose the 10 players without choosing a woman, since there are exactly 10 men. Therefore (using part (a)) there are $286 - 1 = 285$ ways to choose the players if at least one of them must be a woman.

34. Probably the best way to do this is just to break it down into the three cases by sex. There are $C(15,6)$ ways to choose the committee to be composed only of women, $C(15,5)C(10,1)$ ways if there are to be five women and one man, and $C(15,4)C(10,2)$ ways if there are to be four women and two men. Therefore the answer is $C(15,6) + C(15,5)C(10,1) + C(15,4)C(10,2) = 5005 + 30030 + 61425 = 96{,}460$.

38. $C(45,3) \cdot C(57,4) \cdot C(69,5) = 14190 \cdot 395010 \cdot 11238513 \approx 6.3 \times 10^{16}$