CS441 – Discrete Structures for Computer Science
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Problem from Section 5.4

6. $\binom{11}{7}1^4 = 330$

Problem from Section 6.1

6. There are 16 cards that qualify as being an ace or a heart, so the answer is $16/52 = 4/13 \approx 0.31$. We could also compute this from Theorem 2 as $4/52 + 13/52 - 1/52$.

8. We saw in Example 11 of Section 5.3 that there are $C(52,5)$ possible poker hands, and we assume by symmetry that they are all equally likely. In order to solve this problem, we need to compute the number of poker hands that contain the ace of hearts. There is no choice about choosing the ace of hearts. To form the rest of the hand, we need to choose 4 cards from the 51 remaining cards, so there are $C(51,4)$ hands containing the ace of hearts. Therefore the answer to the question is the ratio

$$\frac{C(51,4)}{C(52,5)} = \frac{5}{52} \approx 9.6\%.$$

The problem can also be done by subtracting from 1 the answer to Exercise 9, since a hand contains the ace of hearts if and only if it is not the case that it does not contain the ace of hearts.

14. We saw in Example 11 of Section 5.3 that there are $C(52,5) = 2{,}598{,}960$ different hands, and we assume by symmetry that they are all equally likely. We need to count the number of hands that have 5 different kinds (ranks). There are $C(13,5)$ ways to choose the kinds. For each card, there are then 4 ways to choose the suit. Therefore there are $C(13,5)\cdot 4^5 = 1{,}317{,}888$ ways to choose the hand. Thus the probability is $1317888/2598960 = 2112/4165 \approx 0.51$.

24. In each case, if the numbers are chosen from the integers from 1 to n, then there are $C(n,6)$ possible entries only one of which is the winning one, so the answer is $1/C(n,6)$.
   a) $1/C(30,6) = 1/593775 \approx 1.7\times 10^{-6}$
   b) $1/C(36,6) = 1/1947792 \approx 5.1\times 10^{-7}$
   c) $1/C(42,6) = 1/5245786 \approx 1.9\times 10^{-7}$
   d) $1/C(48,6) = 1/12271512 \approx 8.1\times 10^{-8}$

32. The number of ways for the drawing to turn out is $100\cdot 99\cdot 98$. The number of ways of ways for the drawing to cause Kumar, Janice, and Pedro each to win a prize is $3\cdot 2\cdot 1$ (three ways for one of these to be picked to win first prize, two ways for one of the others to win second prize, one way for the third to win third prize). Therefore the probability we seek is $(3\cdot 2\cdot 1)/(100\cdot 99\cdot 98) = 1/161700$.