CS441 – Discrete Structures for Computer Science
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Problem from Section 5.1

12. We use the sum rule, adding the number of bit strings of each length up to 6. If we include the empty string, then we get $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 2^7 - 1 = 127$ (using the formula for the sum of a geometric progression—see Example 4 in Section 4.1).

16. We can subtract from the number of strings of length 4 of lower case letters the number of strings of length 4 of lower case letters other than x. Thus the answer is $26^4 - 25^4 = 66{,}351$.

24. a) There are 10 ways to choose the first digit, 9 ways to choose the second, and so on: therefore the answer is $10 \cdot 9 \cdot 8 \cdot 7 = 5040$.

b) There are 10 ways to choose each of the first three digits and 5 ways to choose the last; therefore the answer is $10^3 \cdot 5 = 5000$.

c) There are 4 ways to choose the position that is to be different from 9, and 9 ways to choose the digit to go there. Therefore there are $4 \cdot 9 = 36$ such strings.

30. **a)** By the product rule, the answer is $26^8 = 208{,}827{,}064{,}576$.

b) By the product rule, the answer is $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 62{,}990{,}928{,}000$.

c) This is the same as part **(a)**, except that there are only seven slots to fill, so the answer is $26^7 = 8{,}031{,}810{,}176$.

d) This is similar to **(b)**, except that there is only one choice in the first slot, rather than 26, so the answer is $1 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 2{,}422{,}728{,}000$.

e) This is the same as part **(c)**, except that there are only six slots to fill, so the answer is $26^6 = 308{,}915{,}776$.

f) This is the same as part **(e)**; again there are six slots to fill, so the answer is $26^6 = 308{,}915{,}776$.

g) This is the same as part **(f)**, except that there are only four slots to fill, so the answer is $26^4 = 456{,}976$. We are assuming that the question means that the legal strings are BO????BO, where any letters can fill the middle four slots.

h) By part **(f)**, there are 26^6 strings that start with the letters BO in that order. By the same argument, there are 26^6 strings that end that way. By part **(g)**, there are 26^4 strings that both start and end with the letters BO in that order. Therefore by the inclusion–exclusion principle, the answer is $26^6 + 26^6 - 26^4 = 617{,}374{,}576$.

42. There are 2^5 strings that begin with two 0's (since there are two choices for each of the last five bits). Similarly there are 2^4 strings that end with three 1's. Furthermore, there are 2^2 strings that both begin with two 0's and end with three 1's (since only bits 3 and 4 are free to be chosen). By the inclusion–exclusion principle there are $2^5 + 2^4 - 2^2 = 44$ such strings in all.