Problems from Section 4.1

20. The basis step is $n = 7$, and indeed $3^7 < 7!$, since $2187 < 5040$. Assume the statement for k. Then $3^{k+1} = 3 \cdot 3^k < (k+1) \cdot 3^k < (k+1) \cdot k! = (k+1)!$, the statement for $k+1$.

Problems from Section 4.2

4. a) $P(18)$ is true, because we can form 18 cents of postage with one 4-cent stamp and two 7-cent stamps. $P(19)$ is true, because we can form 19 cents of postage with three 4-cent stamps and one 7-cent stamp. $P(20)$ is true, because we can form 20 cents of postage with five 4-cent stamps. $P(21)$ is true, because we can form 20 cents of postage with three 7-cent stamps.

b) The inductive hypothesis is the statement that using just 4-cent and 7-cent stamps we can form j cents postage for all j with $18 \leq j \leq k$, where we assume that $k \geq 21$.

c) In the inductive step we must show, assuming the inductive hypothesis, that we can form $k+1$ cents postage using just 4-cent and 7-cent stamps.

d) We want to form $k+1$ cents of postage. Since $k \geq 21$, we know that $P(k-3)$ is true, that is, that we can form $k-3$ cents of postage. Put one more 4-cent stamp on the envelope, and we have formed $k+1$ cents of postage, as desired.

e) We have completed both the basis step and the inductive step, so by the principle of strong induction, the statement is true for every integer n greater than or equal to 18.