CS441 – Discrete Structures for Computer Science
Instructor: Dr.Litman

Problem from Section 4.1

18. a) Plugging in $n = 2$, we see that $P(2)$ is the statement $2! < 2^2$.
b) Since $2! = 2$, this is the true statement $2 < 4$.
c) The inductive hypothesis is the statement that $k! < k^k$.
d) For the inductive step, we want to show for each $k \geq 2$ that $P(k)$ implies $P(k-1)$. In other words, we want to show that assuming the inductive hypothesis (see part (c)) we can prove that $(k+1)! < (k+1)^{k+1}$.
e) $(k+1)! = (k-1)k! < (k+1)k^k < (k+1)(k+1)^k = (k+1)^{k+1}$
f) We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer n greater than 1.

32. The statement is true for the base case, $n =$ [illegible], since $3 \mid 3$. Suppose that $3 \mid (k^3 + 2k)$. We must show that $3 \mid ((k+1)^3 + 2(k+1))$. If we expand the expression in question, we obtain $k^3 + 3k^2 + 3k + 1 + 2k + 2 = (k^3 + 2k) + 3(k^2 + k + 1)$. By the inductive hypothesis, 3 divides $k^3 + 2k$, and certainly 3 divides $3(k^2 + k + 1)$, so 3 divides their sum, and we are done.