CS441 – Discrete Structures for Computer Science
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Problems from Section 1.6

26. We need to prove two things, since this is an "if and only if" statement. First let us prove directly that if n is even then $7n + 4$ is even. Since n is even, it can be written as $2k$ for some integer k. Then $7n + 4 = 14k + 4 = 2(7k + 2)$. This is 2 times an integer, so it is even, as desired. Next we give a proof by contraposition that if $7n + 4$ is even then n is even. So suppose that n is not even, i.e., that n is odd. Then n can be written as $2k + 1$ for some integer k. Thus $7n + 4 = 14k + 11 = 2(7k + 5) + 1$. This is 1 more than 2 times an integer, so it is odd. That completes the proof by contraposition.

Problems from Section 1.7

10. Of these three numbers, at least two must have the same sign (both positive or both negative), since there are only two signs. (It is conceivable that some of them are zero, but we view zero as positive for the purposes of this problem.) The product of two with the same sign is nonnegative. This was a nonconstructive proof, since we have not identified which product is nonnegative. (In fact, a computer algebra system will tell us that all three are positive, so all three products are positive.)

20. We follow the hint. The square of every real number is nonnegative, so $(x - 1/x)^2 \geq 0$. Multiplying this out and simplifying, we obtain $x^2 - 2 + 1/x^2 \geq 0$, so $x^2 + 1/x^2 \geq 2$, as desired.