CS441 – Discrete Structures for Computer Science
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Problems from Section 1.4

26. a) This is false, since $1+1 \neq 1-1$. b) This is true, since $2+0=2-0$.
c) This is false, since there are many values of y for which $1+y \neq 1-y$.
d) This is false, since the equation $x+2=x-2$ has no solution.
e) This is true, since we can take $x=y=0$. f) This is true, since we can take $y=0$ for each x.
g) This is true, since we can take $y=0$. h) This is false, since part (d) was false.
i) This is certainly false.

30. We need to use the transformations shown in Table 2 of Section 1.3, replacing $\neg\forall$ by $\exists\neg$, and replacing $\neg\exists$ by $\forall\neg$. In other words, we push all the negation symbols inside the quantifiers, changing the sense of the quantifiers as we do so, because of the equivalences in Table 2 of Section 1.3. In addition, we need to use De Morgan's laws (Section 1.2) to change the negation of a conjunction to the disjunction of the negations and to change the negation of a disjunction to the conjunction of the negations. We also use the fact that $\neg\neg p \equiv p$.
a) $\forall y \forall x \neg P(x,y)$ b) $\exists x \forall y \neg P(x,y)$ c) $\forall y(\neg Q(y) \vee \exists x\, R(x,y))$
d) $\forall y(\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))$ e) $\forall y(\exists x \forall z \neg T(x,y,z) \wedge \forall x \exists z \neg U(x,y,z))$

40. a) There are many counterexamples. If $x=2$, then there is no y among the integers such that $2=1/y$, since the only solution of this equation is $y=1/2$. Even if we were working in the domain of real numbers, $x=0$ would provide a counterexample, since $0=1/y$ for no real number y.
b) We can rewrite $y^2-x<100$ as $y^2<100+x$. Since squares can never be negative, no such y exists if x is, say, -200. This x provides a counterexample.
c) This is not true, since sixth powers are both squares and cubes. Trivial counterexamples would include $x=y=0$ and $x=y=1$, but we can also take something like $x=27$ and $y=9$, since $27^2=3^6=9^3$.