Problems from Section 1.3

2. **a)** This is true, since there is an *a* in *orange*. **b)** This is false, since there is no *a* in *lemon*.
c) This is false, since there is no *a* in *true*. **d)** This is true, since there is an *a* in *false*.

6. The answers given here are not unique, but care must be taken not to confuse nonequivalent sentences. Parts **(c)** and **(f)** are equivalent; and parts **(d)** and **(e)** are equivalent. But these two pairs are not equivalent to each other.

a) Some student in the school has visited North Dakota. (Alternatively, there exists a student in the school who has visited North Dakota.)

b) Every student in the school has visited North Dakota. (Alternatively, all students in the school have visited North Dakota.)

c) This is the negation of part **(a)**: No student in the school has visited North Dakota. (Alternatively, there does not exist a student in the school who has visited North Dakota.)

d) Some student in the school has not visited North Dakota. (Alternatively, there exists a student in the school who has not visited North Dakota.)

e) This is the negation of part **(b)**: It is not true that every student in the school has visited North Dakota. (Alternatively, not all students in the school have visited North Dakota.)

f) All students in the school have not visited North Dakota. (This is technically the correct answer, although common English usage takes this sentence to mean—incorrectly—the answer to part **(e)**. To be perfectly clear, one could say that every student in this school has failed to visit North Dakota, or simply that no student has visited North Dakota.)

10. **a)** We assume that this means that one student has all three animals: $\exists x(C(x) \wedge D(x) \wedge F(x))$.
b) $\forall x(C(x) \vee D(x) \vee F(x))$ **c)** $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$
d) This is the negation of part **(a)**: $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$.
e) Here the owners of these pets can be different: $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$.

12. **a)** Since $0 + 1 > 2 \cdot 0$, we know that $Q(0)$ is true.
b) Since $(-1) + 1 > 2 \cdot (-1)$, we know that $Q(-1)$ is true.
c) Since $1 + 1 \not> 2 \cdot 1$, we know that $Q(1)$ is false.
d) From part **(a)** we know that there is at least one x that makes $Q(x)$ true, so $\exists x\, Q(x)$ is true.
e) From part **(c)** we know that there is at least one x that makes $Q(x)$ false, so $\forall x\, Q(x)$ is false.
f) From part **(c)** we know that there is at least one x that makes $Q(x)$ false, so $\exists x\, \neg Q(x)$ is true.
g) From part **(a)** we know that there is at least one x that makes $Q(x)$ true, so $\forall x\, \neg Q(x)$ is false.

14. **a)** Since $(-1)^3 = -1$, this is true.
b) Since $(\frac{1}{2})^4 < (\frac{1}{2})^2$, this is true.
c) Since $(-x)^2 = ((-1)x)^2 = (-1)^2x^2 = x^2$, we know that $\forall x((-x)^2 = x^2)$ is true.
d) Twice a positive number is larger than the number, but this inequality is not true for negative numbers or 0. Therefore $\forall x(2x > x)$ is false.