

Problems from Section 1.2

b)

$$\begin{aligned}
& [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r) \\
& \equiv [(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (\neg p \vee r) && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7}) \\
& \equiv \neg [(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r) && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7}) \\
& \equiv [(\neg p \wedge \neg q) \vee (q \wedge \neg r)] \vee \neg p \vee r && (\text{DeMorgan's law}) \\
& \equiv (\neg p \vee (\neg p \wedge \neg q)) \vee (r \vee (q \wedge \neg r)) && (\text{Associative/ Commutative Law}) \\
& \equiv ((\neg p \vee p) \wedge (\neg p \vee \neg q)) \vee (r \vee q) \wedge (r \vee \neg r) && (\text{Distributive Law}) \\
& \equiv (T \wedge (\neg p \vee \neg q)) \vee (r \vee q) \wedge T && (\text{Negation Law}) \\
& \equiv (\neg p \vee \neg q) \vee (r \vee q) && (\text{Identity Law}) \\
& \equiv \neg p \vee r \vee (q \vee \neg q) && (\text{Commutative/Associative Law}) \\
& \equiv (\neg p \vee r) \vee T && (\text{Identity Law}) \\
& \equiv T && (\text{Domination Law})
\end{aligned}$$

c)

$$\begin{aligned}
& [p \wedge (p \rightarrow q)] \rightarrow q \\
& \equiv [(\neg p) \vee (\neg(\neg p \vee q))] \vee q && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7}) \\
& \equiv [(\neg p) \vee q] \vee (\neg(\neg p \vee q)) && (\text{Commutative/Associative Law}) \\
& \equiv (\neg p \vee q) \vee (\neg(\neg p \vee q)) && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7}) \\
& \equiv T && (\text{Negation Law})
\end{aligned}$$

d)

$$\begin{aligned}
& [(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \\
& \equiv [(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r)] \vee r && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7 and DeMorgan's law}) \\
& \equiv [(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee [(q \vee r) \wedge (\neg r \vee r)] && (\text{Associative and Distributive Law}) \\
& \equiv [(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee [(q \vee r) \wedge T] && (\text{Negation Law}) \\
& \equiv [(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee (q \vee r) && (\text{Identity Law}) \\
& \equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee r \vee q && (\text{Commutative Law}) \\
& \equiv (\neg p \wedge \neg q) \vee q \vee ((p \wedge \neg r) \vee r) && (\text{Associative Laws}) \\
& \equiv ((q \vee \neg p) \wedge (q \vee \neg q)) \vee ((r \vee p) \wedge (r \vee \neg r)) && (\text{Commutative/Distributive Law}) \\
& \equiv ((q \vee \neg p) \wedge T) \vee ((r \vee p) \wedge T) && (\text{Negation Law}) \\
& \equiv (q \vee \neg p) \vee r \vee p && (\text{Identity Law}) \\
& \equiv (q \vee r) \vee (\neg p \vee p) && (\text{Associative Laws}) \\
& \equiv (q \vee r) \vee T && (\text{Negation Law}) \\
& \equiv T && (\text{Domination Law})
\end{aligned}$$