CS441 – Discrete Structures for Computer Science
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Problems from Section 1.3

36. a) Since $1^2 = 1$, this statement is false: $x = 1$ is a counterexample. So is $x = 0$ (these are the only two counterexamples).

b) There are two counterexamples: $x = \sqrt{2}$ and $x = -\sqrt{2}$.

c) There is one counterexample: $x = 0$.

Problems from Section 1.4

2. a) There exists a real number x such that for every real number y, $xy = y$. This is asserting the existence of a multiplicative identity for the real numbers, and the statement is true, since we can take $x = 1$.

b) For every real number x and real number y, if x is nonnegative and y is negative, then the difference $x - y$ is positive. Or, more simply, a nonnegative number minus a negative number is positive (which is true).

c) For every real number x and real number y, there exists a real number z such that $x = y + z$. This is a true statement, since we can take $z = x - y$ in each case.

8. a) $\exists x \exists y Q(x, y)$

b) This is the negation of part (a), and so could be written either $\neg\exists x \exists y Q(x, y)$ or $\forall x \forall y \neg Q(x, y)$.

c) We assume from the wording that the statement means that the same person appeared on both shows: $\exists x(Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of Fortune}))$

d) $\forall y \exists x Q(x, y)$ e) $\exists x_1 \exists x_2 (Q(x_1, \text{Jeopardy}) \wedge Q(x_2, \text{Jeopardy}) \wedge x_1 \neq x_2)$