CS441 – Discrete Structures for Computer Science
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Problems from Section 1.3

20. Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions. See Examples 11 and 16.

a) We want to assert that $P(x)$ is true for some x in the domain, so either $P(-5)$ is true or $P(-3)$ is true or $P(-1)$ is true or $P(1)$ is true or $P(3)$ is true or $P(5)$ is true. Thus the answer is $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$.

b) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

c) The formal translation is as follows: $((-5 \neq 1) \rightarrow P(-5)) \wedge ((-3 \neq 1) \rightarrow P(-3)) \wedge ((-1 \neq 1) \rightarrow P(-1)) \wedge ((1 \neq 1) \rightarrow P(1)) \wedge ((3 \neq 1) \rightarrow P(3)) \wedge ((5 \neq 1) \rightarrow P(5))$. However, since the hypothesis $x \neq 1$ is false when x is 1 and true when x is anything other than 1, we have more simply $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$.

d) The formal translation is as follows: $((-5 \geq 0) \wedge P(-5)) \vee ((-3 \geq 0) \wedge P(-3)) \vee ((-1 \geq 0) \wedge P(-1)) \vee ((1 \geq 0) \wedge P(1)) \vee ((3 \geq 0) \wedge P(3)) \vee ((5 \geq 0) \wedge P(5))$. Since only three of the x's in the domain meet the condition, the answer is equivalent to $P(1) \vee P(3) \vee P(5)$.

e) For the second part we again restrict the domain: $(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-1) \wedge P(-3) \wedge P(-5))$. This is equivalent to $(\neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-1) \wedge P(-3) \wedge P(-5))$.

24. In order to do the translation the second way, we let $C(x)$ be the propositional function "x is in your class." Note that for the second way, we always want to use conditional statements with universal quantifiers and conjunctions with existential quantifiers.

a) Let $P(x)$ be "x has a cellular phone." Then we have $\forall x\, P(x)$ the first way, or $\forall x(C(x) \rightarrow P(x))$ the second way.

b) Let $F(x)$ be "x has seen a foreign movie." Then we have $\exists x\, F(x)$ the first way, or $\exists x(C(x) \wedge F(x))$ the second way.

c) Let $S(x)$ be "x can swim." Then we have $\exists x\, \neg S(x)$ the first way, or $\exists x(C(x) \wedge \neg S(x))$ the second way.

d) Let $Q(x)$ be "x can solve quadratic equations." Then we have $\forall x\, Q(x)$ the first way, or $\forall x(C(x) \rightarrow Q(x))$ the second way.

e) Let $R(x)$ be "x wants to be rich." Then we have $\exists x\, \neg R(x)$ the first way, or $\exists x(C(x) \wedge \neg R(x))$ the second way.

32. In each case we need to specify some propositional functions (predicates) and identify the domain of discourse.

a) Let $F(x)$ be "x has fleas," and let the domain of discourse be dogs. Our original statement is $\forall x\, F(x)$. Its negation is $\exists x\, \neg F(x)$. In English this reads "There is a dog that does not have fleas."

b) Let $H(x)$ be "x can add," where the domain of discourse is horses. Then our original statement is $\exists x\, H(x)$. Its negation is $\forall x\, \neg H(x)$. In English this is rendered most simply as "No horse can add."

c) Let $C(x)$ be "x can climb," and let the domain of discourse be koalas. Our original statement is $\forall x\, C(x)$. Its negation is $\exists x\, \neg C(x)$. In English this reads "There is a koala that cannot climb."

d) Let $F(x)$ be "x can speak French," and let the domain of discourse be monkeys. Our original statement is $\neg \exists x\, F(x)$ or $\forall x\, \neg F(x)$. Its negation is $\exists x\, F(x)$. In English this reads "There is a monkey that can speak French."

e) Let $S(x)$ be "x can swim" and let $C(x)$ be "x can catch fish," where the domain of discourse is pigs. Then our original statement is $\exists x\, (S(x) \wedge C(x))$. Its negation is $\forall x\, \neg(S(x) \wedge C(x))$, which could also be written $\forall x\, (\neg S(x) \vee \neg C(x))$ by De Morgan's law. In English this is "No pig can both swim and catch fish," or "Every pig either is unable to swim or is unable to catch fish."