CS441 – Discrete Structures for Computer Science
Instructor: Dr.Litman

Problems from Section 1.2

14. This is not a tautology. It is saying that knowing that the hypothesis of an conditional statement is false allows us to conclude that the conclusion is also false, and we know that this is not valid reasoning. To show that it is not a tautology, we need to find truth assignments for p and q that make the entire proposition false. Since this is possible only if the conclusion if false, we want to let q be true; and since we want the hypothesis to be true, we must also let p be false. It is easy to check that if, indeed, p is false and q is true, then the conditional statement is false. Therefore it is not a tautology.

18. It is easy to see from the definitions of conditional statement and negation that each of these propositions is false in the case in which p is true and q is false, and true in the other three cases. Therefore the two propositions are logically equivalent.

22. Suppose that $(p \rightarrow q) \wedge (p \rightarrow r)$ is true. We want to show that $p \rightarrow (q \wedge r)$ is true, which means that we want to show that $q \wedge r$ is true whenever p is true. If p is true, since we know that both $p \rightarrow q$ and $p \rightarrow r$ are true from our assumption, we can conclude that q is true and that r is true. Therefore $q \wedge r$ is true, as desired. Conversely, suppose that $p \rightarrow (q \wedge r)$ is true. We need to show that $p \rightarrow q$ is true and that $p \rightarrow r$ is true, which means that if p is true, then so are q and r. But this follows from $p \rightarrow (q \wedge r)$.

32. We just need to find an assignment of truth values that makes one of these propositions true and the other false. We can let p be true and the other two variables be false. Then the first statement will be $\mathbf{F} \rightarrow \mathbf{F}$, which is true, but the second will be $\mathbf{F} \wedge \mathbf{T}$, which is false.