CS441 – Discrete Structures for Computer Science
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Problem from Section 6.2

12. Clearly $p(E \cup F) \geq p(E) = 0.8$. Also, $p(E \cup F) \leq 1$. If we apply Theorem 2 from Section 6.1, we can rewrite this as $p(E) + p(F) - p(E \cap F) \leq 1$, or $0.8 + 0.6 - p(E \cap F) \leq 1$. Solving for $p(E \cap F)$ gives $p(E \cap F) \geq 0.4$.

24. There are 16 equally likely outcomes of flipping a fair coin five times in which the first flip comes up tails (each of the other flips can be either heads or tails). Of these only one will result in four heads appearing, namely $THHHH$. Therefore the answer is $1/16$.

26. Intuitively the answer should be yes, because the parity of the number of 1's is a fifty-fifty proposition totally determined by any one of the flips (for example, the last flip). What happened on the other flips is really rather irrelevant. Let us be more rigorous, though. There are 8 bit strings of length 3, and 4 of them contain an odd number of 1's (namely 001, 010, 100, and 111). Therefore $p(E) = 4/8 = 1/2$. Since 4 bit strings of length 3 start with a 1 (namely 100, 101, 110, and 111), we see that $p(F) = 4/8 = 1/2$ as well. Furthermore, since there are 2 strings that start with a 1 and contain an odd number of 1's (namely 100 and 111), we see that $p(E \cap F) = 2/8 = 1/4$. Then since $p(E) \cdot p(F) = (1/2) \cdot (1/2) = 1/4 = p(E \cap F)$, we conclude from the definition that E and F are independent.

30. **a)** The probability that all bits are a 1 is $(1/2)^{10} = 1/1024$. This is what is being asked for.
b) This is the same as part (a), except that the probability of a 1 bit is 0.6 rather than $1/2$. Thus the answer is $0.6^{10} \approx 0.0060$.
c) We need to multiply the probabilities of each bit being a 1, so the answer is

$$\frac{1}{2} \cdot \frac{1}{2^2} \cdots \frac{1}{2^{10}} = \frac{1}{2^{1+2+\cdots+10}} = \frac{1}{2^{55}} \approx 2.8 \times 10^{-17}.$$

Note that this is essentially 0.

38. We assume that the observer was instructed ahead of time to tell us whether or not at least one die came up 6 and to provide no more information than that. If we do not make such an assumption, then the following analysis would not be valid. We use the notation (i, j) to represent that the first die came up i and the second die came up j. Note that there are 36 equally likely outcomes.
a) Let S be the event that at least one die came up 6, and let T be the event that sum of the dice is 7. We want $p(T \mid S)$. By Definition 3, this equals $p(S \cap T)/p(S)$. The outcomes in $S \cap T$ are (1, 6) and (6, 1), so $p(S \cap T) = 2/36$. There are $5^2 = 25$ outcomes in $\overline{S}$ (five ways to choose what happened on each die), so $p(S) = (36 - 25)/36 = 11/36$. Therefore the answer is $(2/36)/(11/36) = 2/11$.