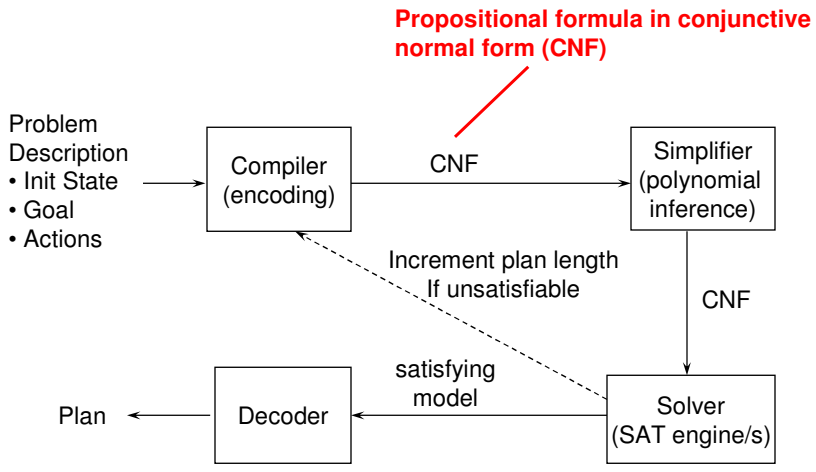
Planning as Satisfiability

- Planning as propositional satisfiability

* Based on slides by Alan Fern, Stuart Russell and Dana Nau

1

Architecture of a SAT-based Planner



2

Propositional Satisfiability

- A formula is ***satisfiable*** if it is true in some model
  - e.g. $A \vee B$, C
- A formula is unsatisfiable if it is true in no models
  - e.g. $A \wedge \neg A$
- Testing satisfiability of CNF formulas is a famous NP-complete problem

3

Propositional Satisfiability

- Many problems (such as planning) can be naturally encoded as instances of satisfiability
- Thus there has been much work on developing powerful satisfiability solvers
  - these solvers work amazingly well in practice (we will touch on some later)

4

Encoding Planning as Satisfiability: Basic Idea

- Bounded planning problem (P,n):
  - P is a planning problem; n is a positive integer
  - Find a solution for P of length n
- Create a propositional formula that represents:
  - Initial state
  - Goal
  - Action Dynamics

  for n time steps
- We will define the formula for (P,n) such that:
  1) ***any*** model (i.e. satisfying truth assignment) of the formula represent a solution to (P,n)
  2) if (P,n) has a solution then the formula is satisfiable

5

Encoding Planning Problems

- We can encode (P,n) so that we consider either layered plans or totally ordered plans
  - an advantage of considering layered plans is that fewer time steps are necessary (i.e. smaller n translates into smaller formulas)
  - for simplicity we first consider totally-ordered plans
- Encode (P,n) as a formula Φ such that

  $\langle a_0, a_1, \ldots, a_{n-1} \rangle$ is a solution for (P,n)

  if and only if

  Φ can be satisfied in a way that makes the fluents $a_0, \ldots, a_{n-1}$ true
- Φ will be conjunction of many other formulas …

6

Formulas in Φ

- Formula describing the **initial state**: (let *E* be the set of possible facts in the planning problem)

  $\bigwedge\{e_0 \mid e \in s_0\} \wedge \bigwedge\{\neg e_0 \mid e \in E - s_0\}$

  Describes the complete initial state (both positive and negative fact)

  - E.g. $on(A,B,0) \wedge \neg on(B,A,0)$

- Formula describing the **goal**: (*G* is set of goal facts)

  $\bigwedge\{e_n \mid e \in G\}$

  says that the goal facts must be true in the final state at timestep *n*

  - E.g. $on(B,A,n)$

- Is this enough?
  - Of course not. The formulas say nothing about actions.

7

Formulas in Φ

- For every action *a* and timestep *i*, formula describing what fluents must be true if *a* were the *i*'th step of the plan:
  - $a_i \Rightarrow \bigwedge\{e_i \mid e \in Precond(a)\}$, *a*'s preconditions must be true
  - $a_i \Rightarrow \bigwedge\{e_{i+1} \mid e \in ADD(a)\}$, a's ADD effects must be true in i+1
  - $a_i \Rightarrow \bigwedge\{\neg e_{i+1} \mid e \in DEL(a)\}$, a's DEL effects must be false in i+1
- *Complete exclusion* axiom:
  - For all actions *a* and *b* and timesteps *i*, formulas saying *a* and *b* can't occur at the same time

    $\neg a_i \vee \neg b_i$
  - this guarantees there can be only one action at a time
- Is this enough?
  - The formulas say nothing about what happens to facts if they are not effected by an action
  - This is known as the ***frame problem***

8

Frame Axioms

- *Frame axioms*:
  - Formulas describing what *doesn't* change between steps i and i+1
- Several ways to write these (your book shows another way)
  - Here I show a alternative that typically works best in practice
- **explanatory frame axioms**
  - One axiom for every possible fact e *at every* timestep i
  - Says that if e changes truth value between s_i and s_{i+1}, then the action at step i must be responsible:

$$\neg e_i \wedge e_{i+1} \Rightarrow \mathrm{V}\{a_i \,/\, \mathrm{e}\ \mathit{in}\ \mathrm{ADD}(a)\}$$

If e became true then some action must have added it

$$e_i \wedge \neg e_{i+1} \Rightarrow \mathrm{V}\{a_i \,/\, \mathrm{e}\ \mathit{in}\ \mathrm{DEL}(a)\}$$

If e became false then some action must have deleted it

9

Example

- Planning domain:
  - one robot r1
  - two adjacent locations l1, l2
  - one operator (move the robot)
- Encode (P,n) where $n = 1$
  - Initial state: {at(r1,l1)}
    Encoding: at(r1,l1,0) ∧ ¬at(r1,l2,0)
  - Goal: {at(r1,l2)}
    Encoding: at(r1,l2,1)
  - Action Schema: see next slide

10

Example (continued)

- Schema: move(r, l, l')
  PRE: at(r,l)
  ADD: at(r,l')
  DEL: at(r,l)

Encoding: (for actions move(r1,l1,l2) and move(r1,l2,l1) at time step 0)

move(r1,l1,l2,0) ⇒ at(r1,l1,0)
move(r1,l1,l2,0) ⇒ at(r1,l2,1)
move(r1,l1,l2,0) ⇒ ¬at(r1,l1,1)

move(r1,l2,l1,0) ⇒ at(r1,l2,0)
move(r1,l2,l1,0) ⇒ at(r1,l1,1)
move(r1,l2,l1,0) ⇒ ¬at(r1,l2,1)

11

Example (continued)

- Schema: move(r, l, l')
  PRE: at(r,l)
  ADD: at(r,l')
  DEL: at(r,l)

- Complete-exclusion axiom:
  ¬move(r1,l1,l2,0) ∨ ¬move(r1,l2,l1,0)

- Explanatory frame axioms:
  ¬at(r1,l1,0) ∧ at(r1,l1,1) ⇒ move(r1,l2,l1,0)
  ¬at(r1,l2,0) ∧ at(r1,l2,1) ⇒ move(r1,l1,l2,0)
  at(r1,l1,0) ∧ ¬at(r1,l1,1) ⇒ move(r1,l1,l2,0)
  at(r1,l2,0) ∧ ¬at(r1,l2,1) ⇒ move(r1,l2,l1,0)

12

Complete Formula for (*P*,1)

$[\ at(r1,l1,0) \wedge \neg at(r1,l2,0)\ ] \wedge$
$at(r1,l2,1) \wedge$
$[\ move(r1,l1,l2,0) \Rightarrow at(r1,l1,0)\ ] \wedge$
$[\ move(r1,l1,l2,0) \Rightarrow at(r1,l2,1)\ ] \wedge$
$[\ move(r1,l1,l2,0) \Rightarrow \neg at(r1,l1,1)\ ] \wedge$
$[\ move(r1,l2,l1,0) \Rightarrow at(r1,l2,0)\ ] \wedge$
$[\ move(r1,l2,l1,0) \Rightarrow at(r1,l1,1)\ ] \wedge$
$[\ move(r1,l2,l1,0) \Rightarrow \neg at(r1,l2,1)\ ] \wedge$
$[\ \neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0)\ ] \wedge$
$[\ \neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)\ ] \wedge$
$[\ \neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0)\ ] \wedge$
$[\ at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0)\ ] \wedge$
$[\ at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)\ ]$

Convert to CNF and give to SAT solver.

13

Extracting a Plan

- Suppose we find an assignment of truth values that satisfies Φ.
  - This means *P* has a solution of length *n*
- For *i*=0,…,*n-1*, there will be exactly one action *a* such that a_i = *true*
  - This is the *i*'th action of the plan.
- Example (from the previous slides):
  - Φ can be satisfied with move(r1,l1,l2,0) = *true*
  - Thus ⟨move(r1,l1,l2,0)⟩ is a solution for (*P*,0)
    - It's the only solution - no other way to satisfy Φ

14

Supporting Layered Plans

- *Complete exclusion* axiom:
  - For **all** actions *a* and *b* and time steps *i* include the formula $\neg a_i \vee \neg b_i$
  - this guaranteed that there could be only one action at a time
- *Partial exclusion axiom*:
  - For any pair of incompatible actions (recall from Graphplan) *a* and *b* and each time step *i* include the formula $\neg a_i \vee \neg b_i$
  - This encoding will allowed for more than one action to be taken at a time step resulting in layered plans
  - This is advantageous because fewer time steps are required (i.e. shorter formulas)

15

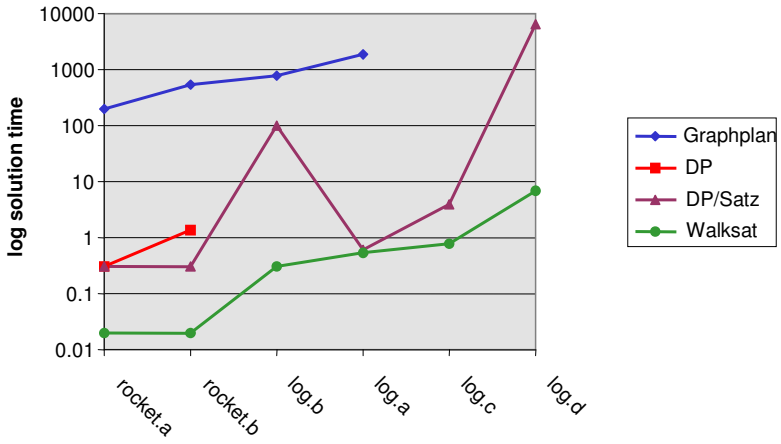
Planning Benchmark Test Set

- Extension of Graphplan test set
- blocks world - up to 18 blocks, 10^{19} states
- logistics - complex, highly-parallel transportation domain.

  Logistics.d:
  - 2,165 possible actions per time slot
  - 10^{16} legal configurations (2^{2000} states)
  - optimal solution contains 74 distinct actions over 14 time slots
- *Problems of this size never previously handled by general-purpose planning systems*

16

Scaling Up Logistics Planning



17

What SATPLAN Shows

- General propositional reasoning can compete with state of the art specialized planning systems
  - New, highly tuned variations of DP surprising powerful
  - Radically new stochastic approaches to SAT can provide very low exponential scaling
- Why does it work?
  - More flexible than forward or backward chaining
  - Randomized algorithms less likely to get trapped along bad paths

18

Discussion

- How well does this work?
  - Created an initial splash but by itself, not very practical without help in choosing good encoding
- However combining SatPlan with planning graphs can overcome this problem

19