Making Complex Decisions
Chapter 17(1-2)

Building a policy

- Specify a solution for any initial state
  - Construct a policy that outputs the best action for any state
    - policy = π
    - policy in state s = $\pi(s)$
  - Complete policy covers all potential input states
  - Optimal policy, π^*, yields the highest expected utility
    - Why expected?
      - Transitions are stochastic

Using a policy

- An agent in state s
  - s is the percept available to agent

    $\pi^*(s)$ outputs an action that maximizes expected utility
- The policy is a description of a simple reflex

Striking a balance

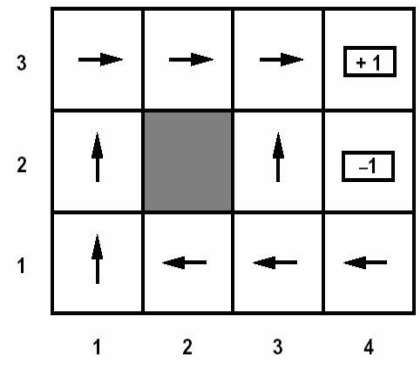
- Different policies demonstrate balance between risk and reward
  - Only interesting in stochastic environments (not deterministic)
  - Characteristic of many real-world problems
- Building the optimal policy is the hard part!

Attributes of optimality

- We wish to find policy that maximizes the utility of agent during lifetime
  - Maximize $U([s_0, s_1, s_2, \ldots, s_n)]$
- But is length of lifetime known?
  - Finite horizon – number of state transitions is known
    - After timestep N, nothing matters
  - $U([s_0, s_1, s_2, \ldots, s_n)] = U([s_0, s_1, s_2, \ldots, s_{n,} s_{n+1}, s_{n+k})]$ for all $k>0$
  - Infinite horizon – always opportunity for more state transitions

Time horizon

- Consider spot (3, 1)
  - Let horizon = 3
  - Let horizon = 8
  - Let horizon = 20
  - Let horizon = inf
  - Does π^* change?



- Nonstationary optimal policy

Evaluating state sequences

- Additive Rewards
  - $U[(a, b, c, \ldots)] = R(a) + R(b) + R(c) + \ldots$
- Discounted Rewards
  - $U[(a, b, c, \ldots)] = R(a) + \gamma R(b) + \gamma^2 R(c) + \ldots$

    γ is the discount factor between 0 and 1
    - What does this mean?

Evaluating a policy

- Each policy, π, generates multiple state sequences
  - Uncertainty in transitions according to T(s, a, s')
- Policy value is an expected sum of discounted rewards observed over all possible state sequences

$$\pi^* = \underset{\pi}{\operatorname{argmax}} E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi\right]$$

Building an optimal policy

- Value Iteration
  - Calculate the utility of each state
  - Use the state utilities to select an optimal action in each state
  - Your policy is simple – go to the state with the best utility
  - Your state utilities must be accurate
    - Through an iterative process you assign correct values to the state utility values

Utility of states

- The utility of a state s is…
  - the expected utility of the state sequences that might follow it
    - The subsequent state sequence is a function of π(s)
- The utility of a state given policy π is…

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi, s_0 = s\right]$$

Restating the policy

- Previous slide said you go to state with highest utility
- Actually…
  - Go to state with maximum expected utility
    - Reachable state with highest utility may have low probability of being obtained
    - Function of available actions, transition function, resulting states

$$\pi^*(s) = \operatorname*{argmax}_a \sum_{s'} T(s,a,s')U(s')$$

Putting pieces together

- We said the utility of a state was:

$$U^\pi(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi, s_0 = s\right]$$

- The policy is maximum expected utility

$$\pi^*(s) = \operatorname*{argmax}_a \sum_{s'} T(s,a,s')U(s')$$

- Therefore, utility of a state is the immediate reward for that state and expected utility of next state

$$U(s) = R(s) + \gamma \max_a \sum_{s'} T(s,a,s')U(s')$$

Example

- Revisit 4x3 example
- Utility at cell (1, 1)

$$U(1,1) = -0.04 + \gamma \max\{ \begin{array}{ll} 0.8U(1,2) + 0.1U(2,1) + 0.1U(1,1), & (Up) \\ 0.9U(1,1) + 0.1U(1,2), & (Left) \\ 0.9U(1,1) + 0.1U(2,1), & (Down) \\ 0.8U(2,1) + 0.1U(1,2) + 0.1U(1,1)\ \} & (Right) \end{array}$$

- Consider all outcomes of all possible actions to select best action and assign its expected utility to value of next-state in equation

Using Bellman Equations to solve MDPs

- Consider a particular MDP
  - *n* possible states
  - *n* Bellman equations (one for each state)
  - *n* equations have *n* unknowns (U(s) for each state)
    - Iterative technique to solve