

Decision-making in the presence of uncertainty

Main issues:

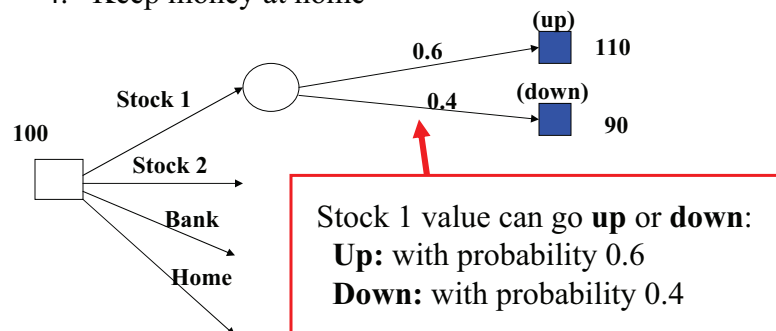
- How to model the decision process with uncertain outcomes in the computer ?
- How to make decisions about actions in the presence of uncertainty?

The field of **decision-making** studies ways of making decisions in the presence of uncertainty.

Decision making example.

Assume we want to invest \$100 for 6 months

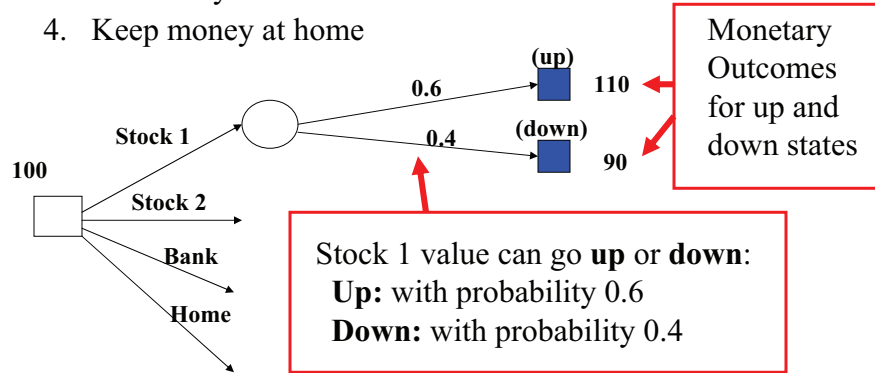
- We have 4 choices:
 1. Invest in Stock 1
 2. Invest in Stock 2
 3. Put money in bank
 4. Keep money at home



Decision making example.

Assume we want to invest \$100 for 6 months

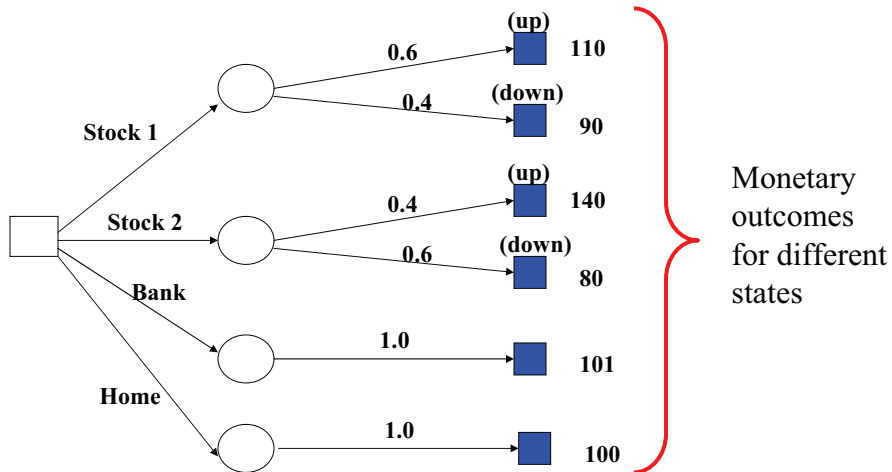
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Decision making example.

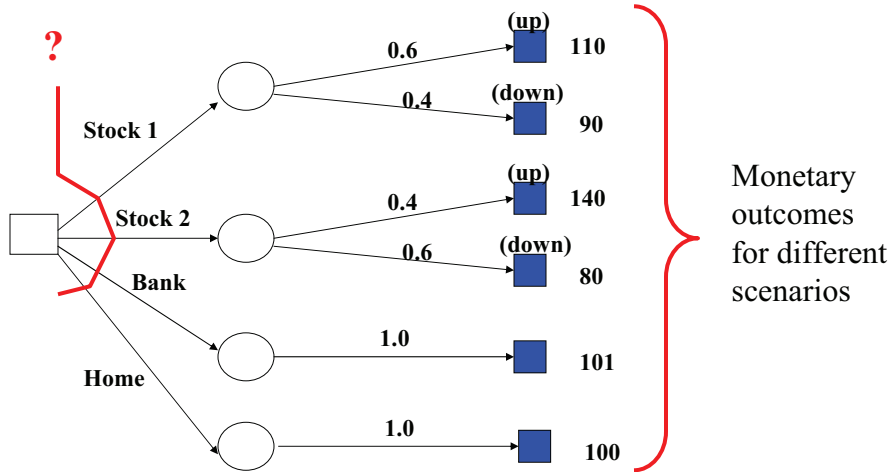
Investing of \$100 for 6 months



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Decision making example.

We need to make a choice whether to invest in Stock 1 or 2, put money into bank or keep them at home. But how?

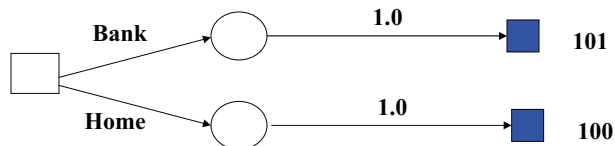


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Decision making example.

Assume the simplified problem with the Bank and Home choices only.

The result is guaranteed – the outcome is deterministic



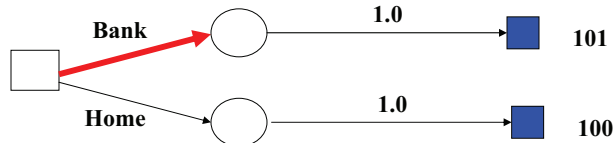
What is the rational choice assuming our goal is to make money?

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Decision making. Deterministic outcome.

Assume the simplified problem with the Bank and Home choices only.

These choices are deterministic.



Our goal is to make money. What is the rational choice?

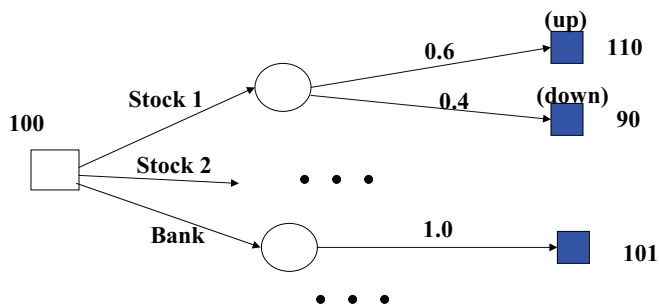
Answer: Put money into the bank. The choice is always strictly better in terms of the outcome

But what to do if we have uncertain outcomes?

Decision making. Stochastic outcome

- How to quantify the goodness of the stochastic outcome?

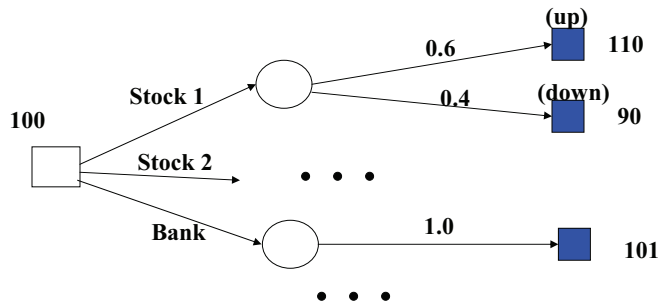
We want to compare it to deterministic and other stochastic outcomes.



?

Decision making. Stochastic outcome

- How to quantify the goodness of the stochastic outcome?
We want to compare it to deterministic and other stochastic outcomes.



Idea: Use expected value of the outcome

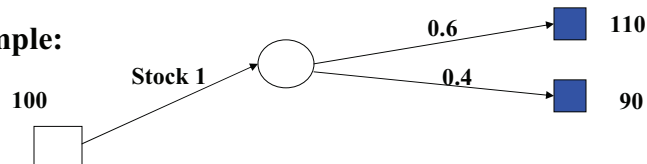
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X .
- **Expected value** of X is:

$$E(X) = \sum_{x \in \Omega_X} xP(X = x)$$

Intuition: Expected value summarizes all stochastic outcomes into a single quantity.

- **Example:**



- What is the expected value of the outcome of Stock 1 option?

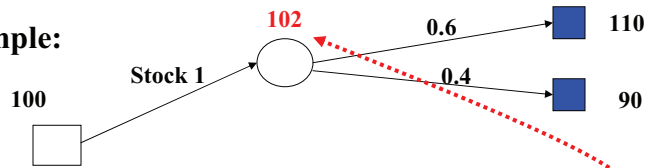
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X .
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- Expected value** summarizes all stochastic outcomes into a single quantity

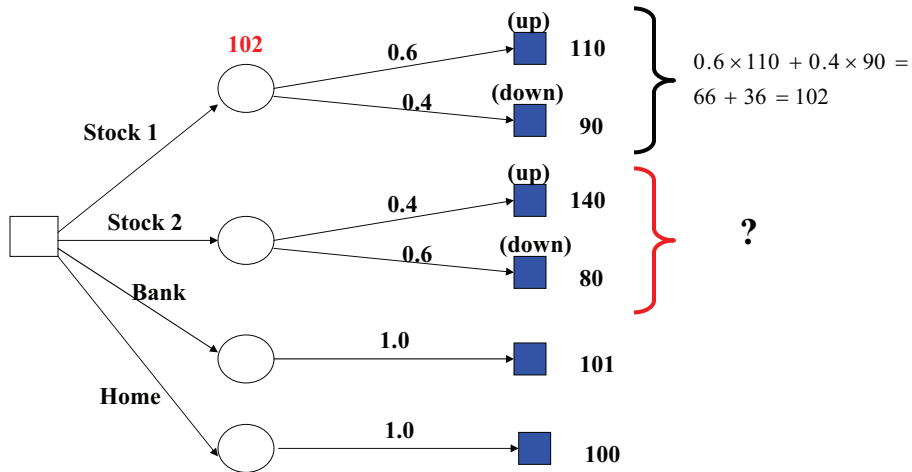
- Example:**



Expected value for the outcome of the Stock 1 option is:
 $0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$

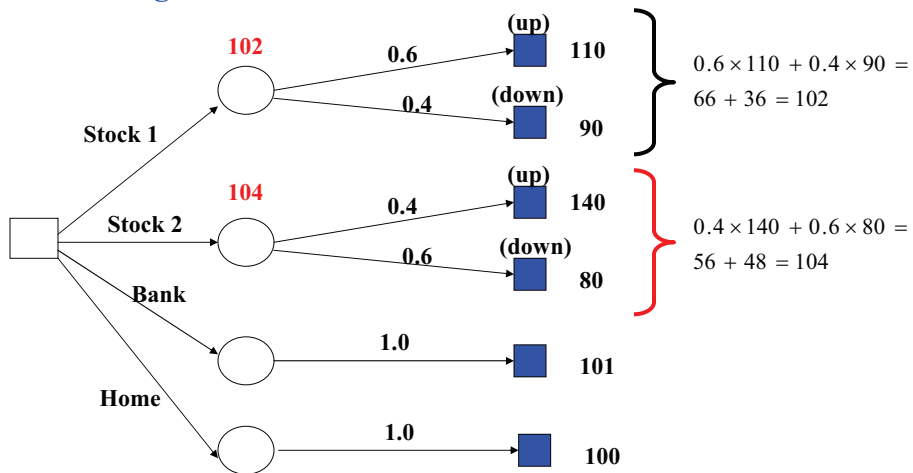
Expected values

Investing \$100 for 6 months



Expected values

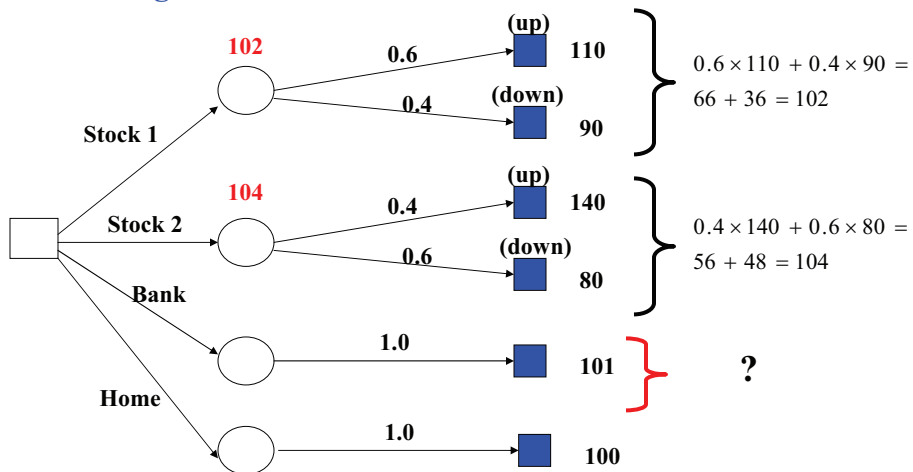
Investing \$100 for 6 months



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Expected values

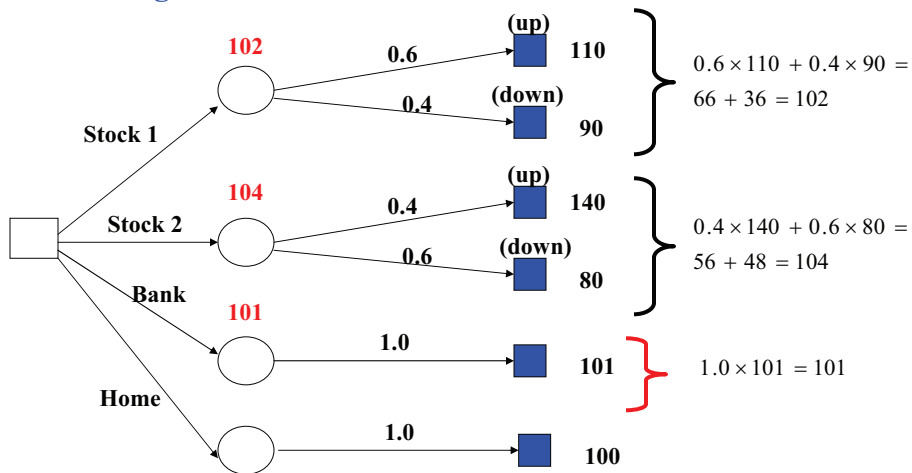
Investing \$100 for 6 months



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Expected values

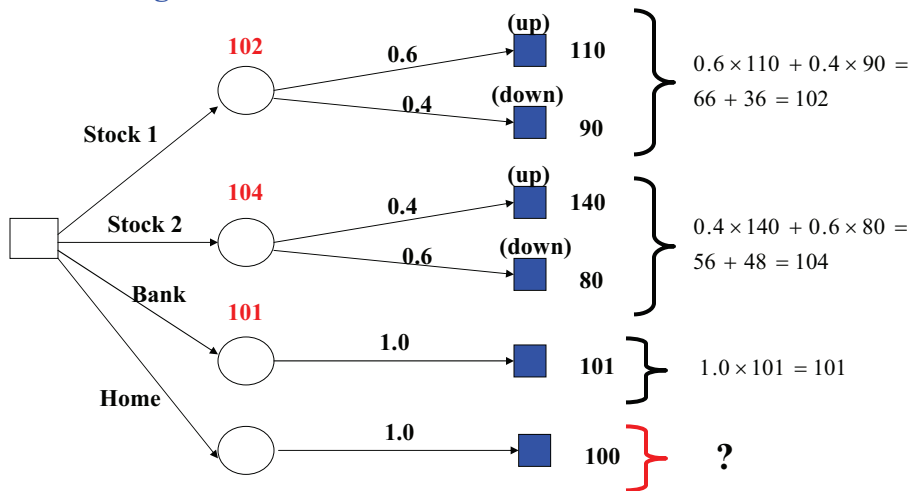
Investing \$100 for 6 months



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Expected values

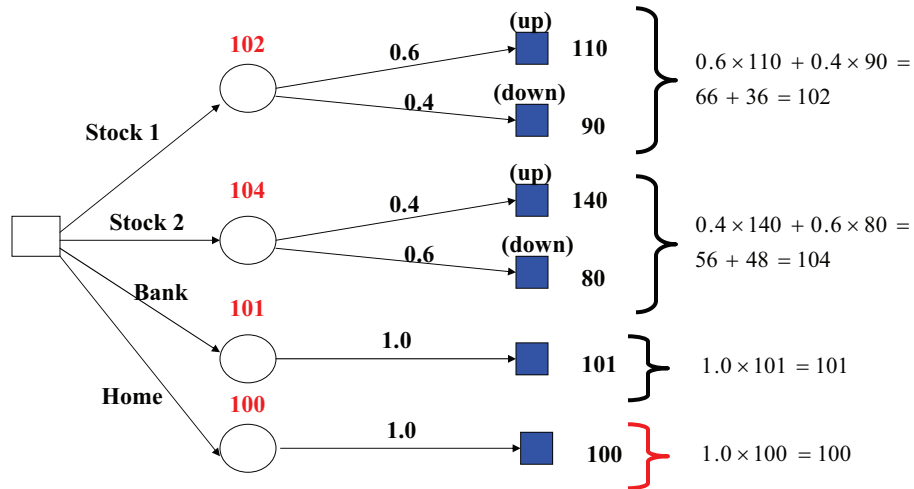
Investing \$100 for 6 months



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Expected values

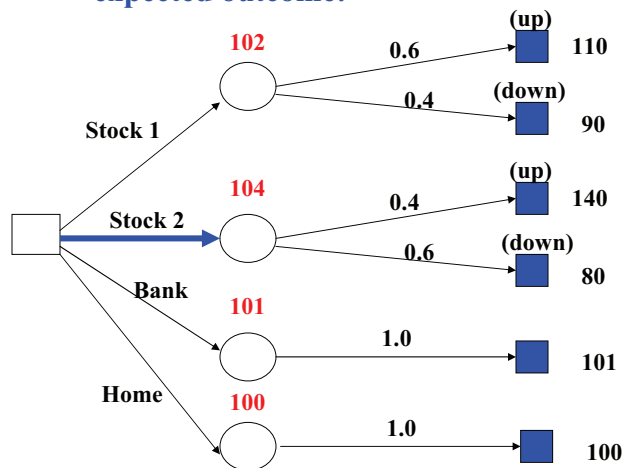
Investing \$100 for 6 months



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Selection based on expected values

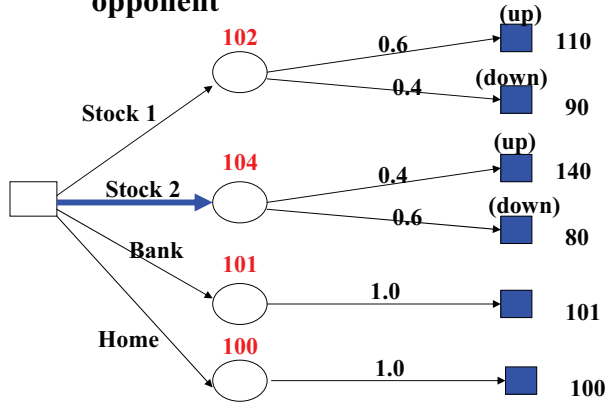
The optimal action is the option that maximizes the expected outcome:



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Relation to the game search

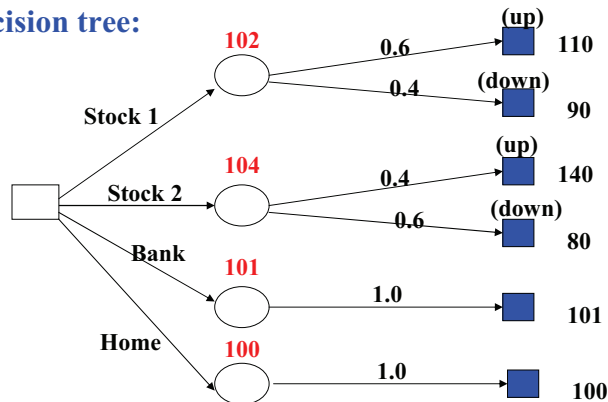
- **Game search: minimax algorithm**
 - considers the rational opponent and its best move
- **Decision making: maximizes the expectation**
 - play against the nature - stochastic non-malicious "opponent"






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(Stochastic) Decision tree

- **Decision tree:**



-  **decision node**
-  **chance node**
-  **outcome (value) node**

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Sequential (multi-step) problems

The decision tree can be built to capture multi-step decision problems:

- Choose an action
- Observe the stochastic outcome
- And repeat

How to make decisions for multi-step problems?

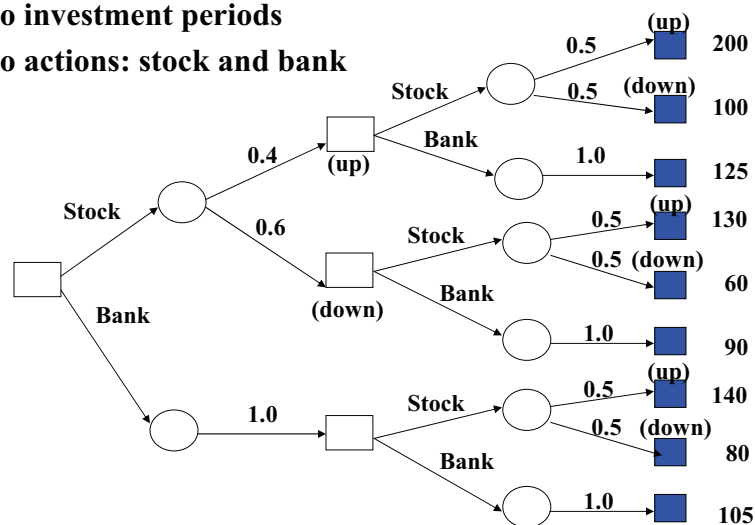
- Start from the leaves of the decision tree (outcome nodes)
- Compute expectations at chance nodes
- Maximize at the decision nodes

Algorithm is sometimes called **expectimax**

Multi-step problem example

Assume:

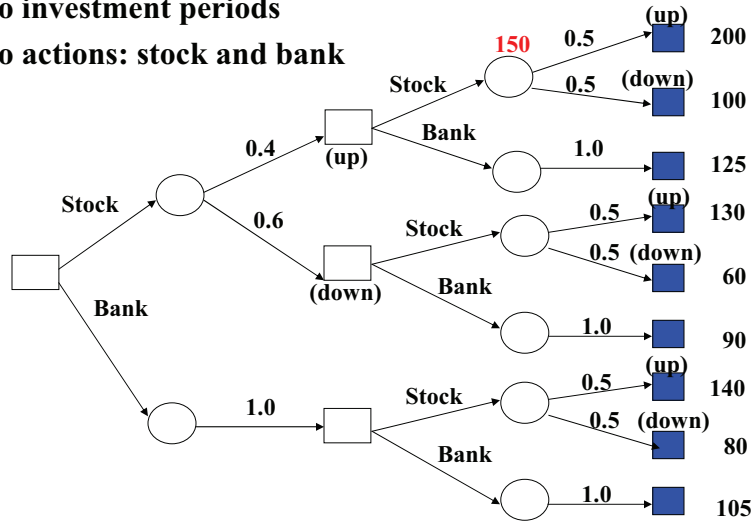
- Two investment periods
- Two actions: stock and bank



Multi-step problem example

Assume:

- Two investment periods
- Two actions: stock and bank

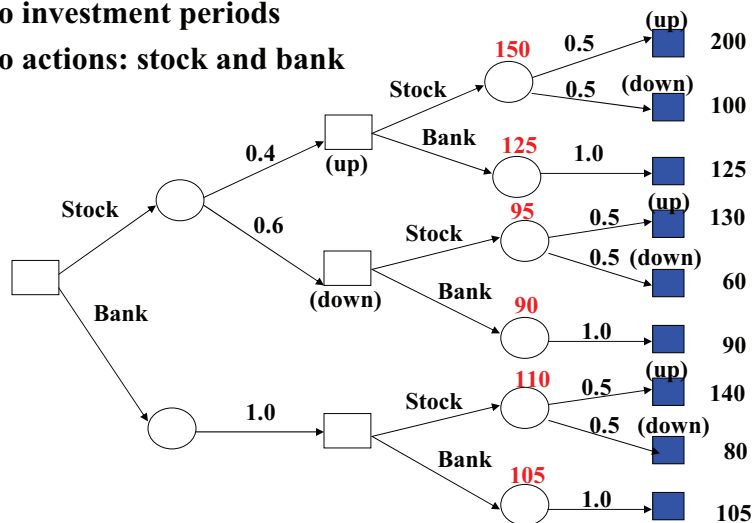


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Multi-step problem example

Assume:

- Two investment periods
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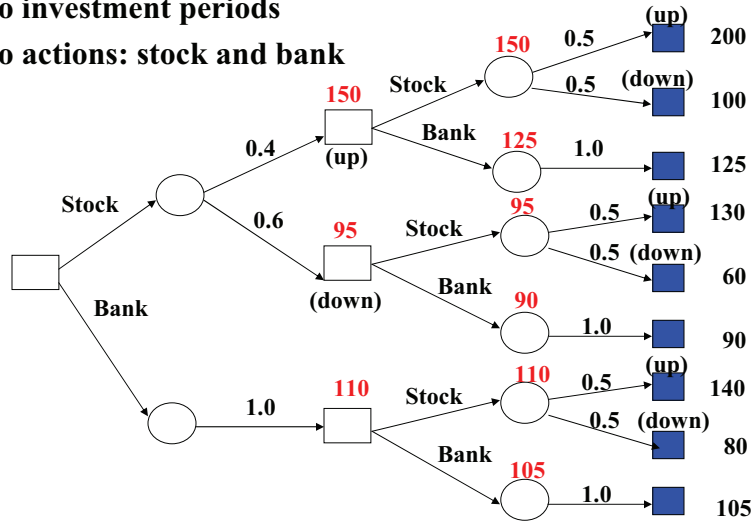


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Multi-step problem example

Assume:

- Two investment periods
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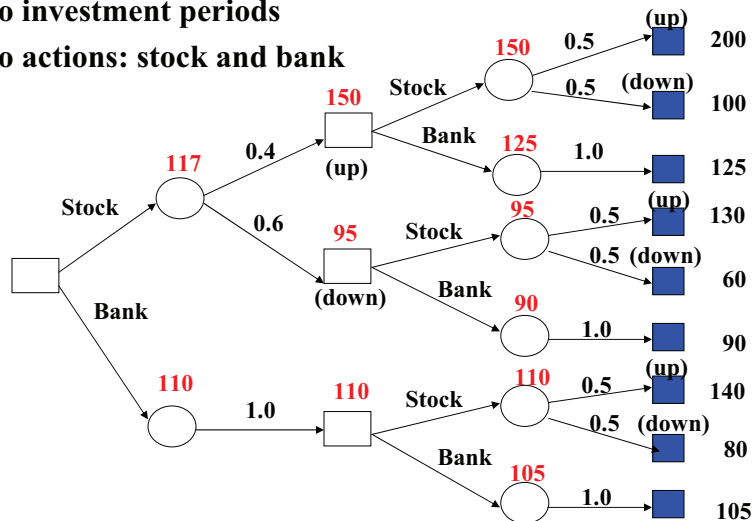


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Multi-step problem example

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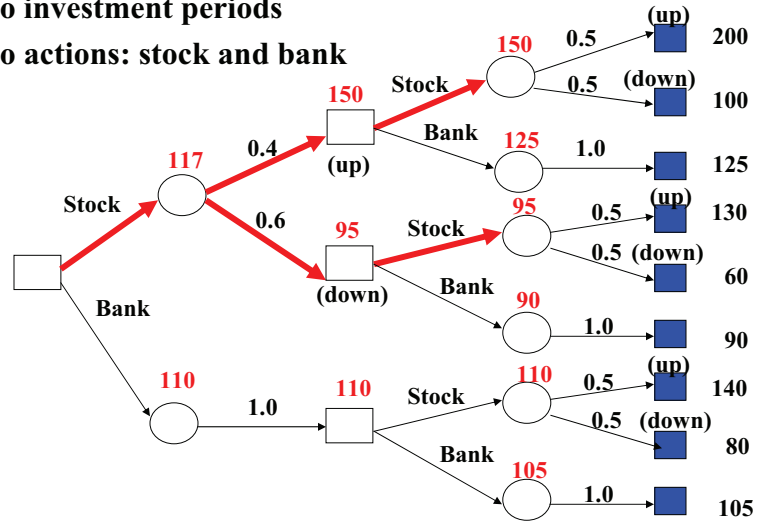


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Multi-step problem example

Assume:

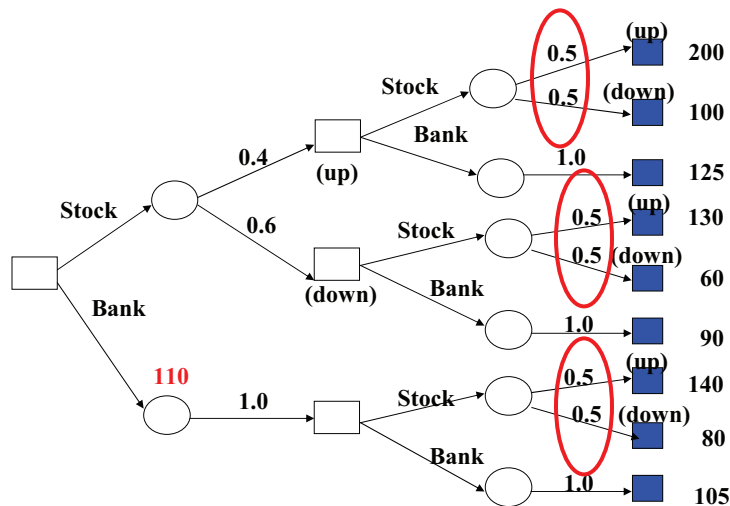
- Two investment periods
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Multi-step problems. Conditioning.

- Notice that the probability of stock going up and down in the 2nd step is independent of the 1st step (=0.5)



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