Informed Search and Exploration

Chapter 4 (4.1-4.3)

Introduction

- Ch.3 searches – good building blocks for learning about search
- But vastly inefficient eg:
- Can we do better?

	Breadth First	Depth First	Uniform Cost
Time	B^D	B^M	$>B^D$ (?)
Space	B^D	BM	$>B^D$ (?)
Optimal?	Y	N	Y
Complete?	Y	N	Y

(Quick Partial) Review

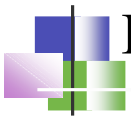
- Previous algorithms differed in how to select next node for expansion eg:
  - Breadth First
    - Fringe nodes sorted old -> new
  - Depth First
    - Fringe nodes sorted new -> old
  - Uniform cost
    - Fringe nodes sorted by path cost: small -> big
- Used little (no) "external" domain knowledge

Overview

- Heuristic Search
  - Best-First Search Approach
    - Greedy
    - A*
  - Heuristic Functions
- Local Search and Optimization
  - Hill-climbing
  - Simulated Annealing
  - Local Beam
  - Genetic Algorithms

Informed Searching

- An *informed search* strategy uses knowledge beyond the definition of the problem
- The knowledge is embodied in an *evaluation function* f(n)



Best-First Search

- An algorithm in which a node is selected for expansion based on an evaluation function f(n)
  - Fringe nodes ordered by f(n)
  - Traditionally the node with the lowest evaluation function is selected
  - Not an accurate name…expanding the best node first would be a straight march to the goal.
  - Choose the node that *appears* to be the best

Best-First Search

- Remember: Uniform cost search
  - F(n) = g(n)
- Best-first search:
  - F(n) = h(n)
- Later, a-star search:
  - F(n) = g(n) + h(n)

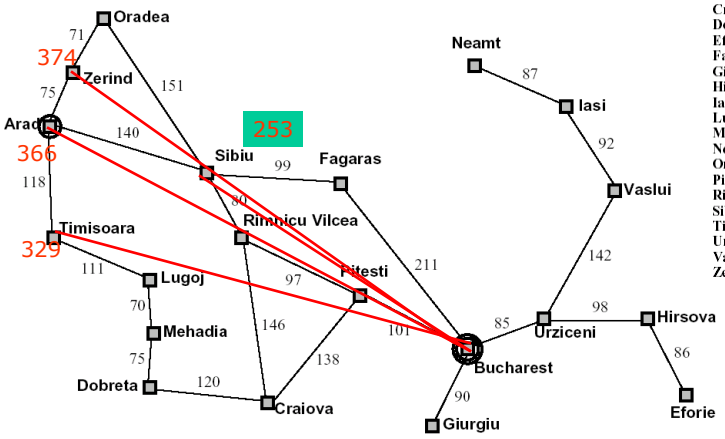
Best-First Search (cont.)

- Some BFS algorithms also include the notion of a heuristic function h(n)
- h(n) = estimated cost of the cheapest path from node n to a goal node
- Best way to include informed knowledge into a search
- Examples:
  - How far is it from point A to point B
  - How much time will it take to complete the rest of the task at current node to finish

Greedy Best-First Search

- Expands node estimated to be closest to the goal
  - f(n) = h(n)
- Consider the route finding problem.
  - Can we use additional information to avoid costly paths that lead nowhere?
  - Consider using the straight line distance (SLD)

Route Finding



Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
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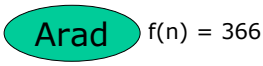
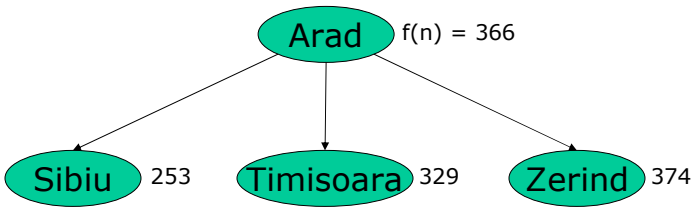
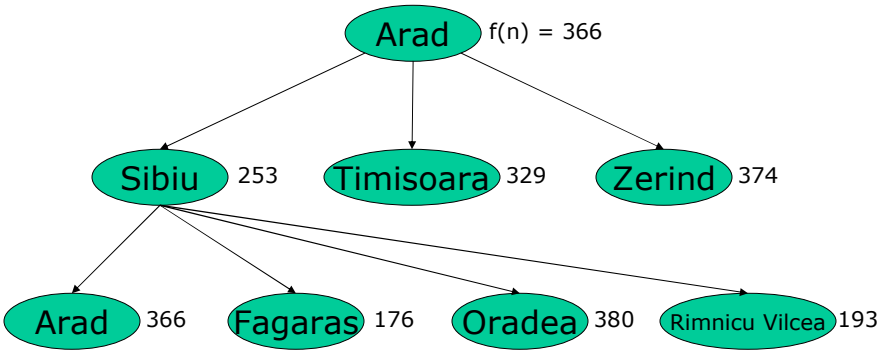
Route Finding: Greedy Best First



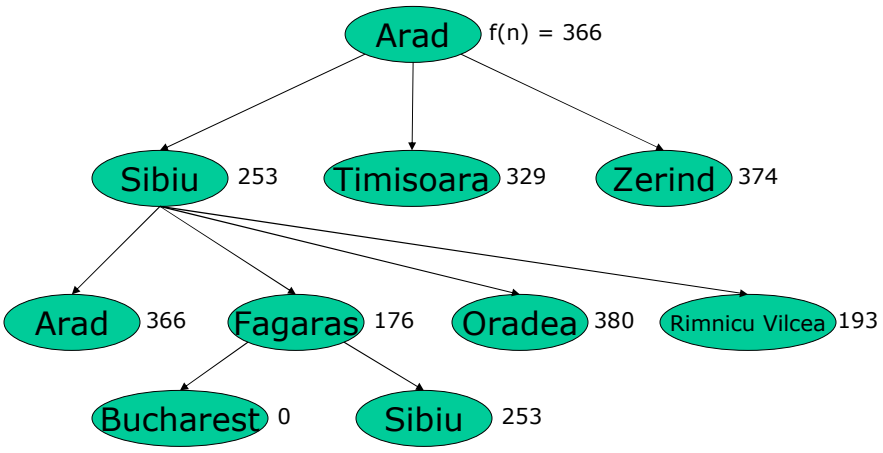
Route Finding: Greedy Best First



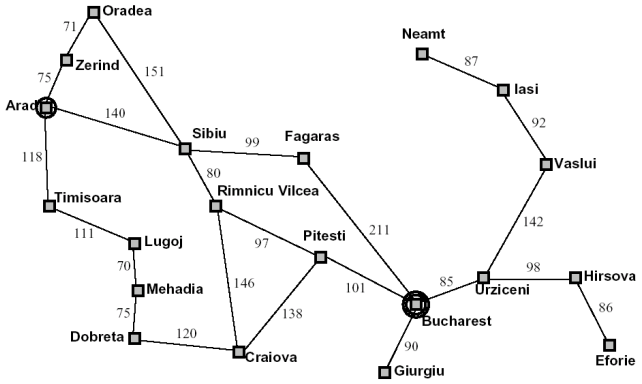
Route Finding: Greedy Best First



Route Finding: Greedy Best First



Exercise



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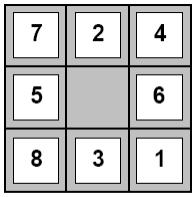
So is Arad->Sibiu->Fagaras->Bucharest optimal?

Greedy Best-First Search

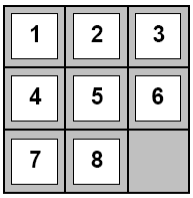
- Not optimal.
- Not complete.
  - Could go down a path and never return to try another.
  - e.g., Iasi → Neamt → Iasi → Neamt → …
- Space Complexity
  - $O(b^m)$ – keeps all nodes in memory
- Time Complexity
  - $O(b^m)$ (but a good heuristic can give a dramatic improvement)

Heuristic Functions

- Example: 8-Puzzle
  - Average solution cost for a random puzzle is 22 moves
  - Branching factor is about 3
    - Empty tile in the middle -> four moves
    - Empty tile on the edge -> three moves
    - Empty tile in corner -> two moves
  - 3^{22} is approx 3.1e10
    - Get rid of repeated states
    - 181,440 distinct states

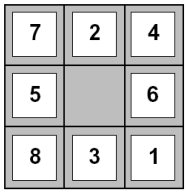


Start State



Goal State

Heuristic Functions



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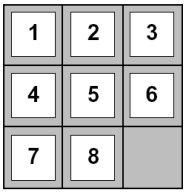
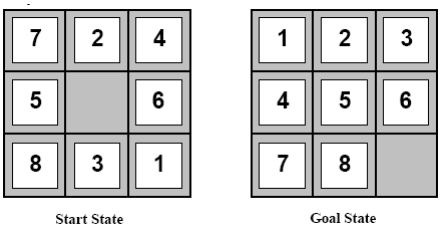


Goal State

- h1 = number of misplaced tiles
- h2 = sum of distances of tiles to goal position.

Heuristic Functions

- h1 = 7
- h2 = 4+0+3+3+1+0+2+1 = 14



Admissible Heuristics

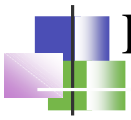
- A heuristic function h(n) is *admissible* if it never overestimates the cost to reach the goal from n
- Another property of heuristic functions is *consistency*
  - $h(n) \leq c(n,a,n') + h(n')$ where:
    - c(n,a,n') is the cost to get to n' from n using action a.
  - Consistent h(n) → the values of f(n) along any path are non-decreasing
- Graph search is optimal if h(n) is consistent

Heuristic Functions

- Is h1 (#of displaced tiles)
  - admissible?
  - consistent?
- Is h2 (Manhattan distance)
  - admissible?
  - consistent?

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
  - then h_2 dominates h_1
  - h_2 is better for search
- Typical search costs (average number of nodes expanded):
- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes □



Heuristic Functions

- Heuristics are often obtained from *relaxed problem*
  - Simplify the original problem by removing constraints
  - The cost of an optimal solution to a relaxed problem is an admissible heuristic.

8-Puzzle

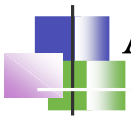
- Original
  - A tile can move from *A* to *B* if *A* is horizontally or vertically **adjacent** to *B* and *B* is **blank**.
- Relaxations
  - Move from *A* to *B* if *A* is adjacent to *B* *(remove "blank")*
    - *h2 by moving each tile in turn to destination*
  - Move from *A* to *B* *(remove "adjacent" and "blank")*
    - *h1 by simply moving each tile directly to destination*

How to Obtain Heuristics?

- Ask the domain expert (if there is one)
- Solve example problems and generalize your experience on which operators are helpful in which situation (particularly important for state space search)
- Try to develop sophisticated evaluation functions that measure the closeness of a state to a goal state (particularly important for state space search)
- Run your search algorithm with different parameter settings trying to determine which parameter settings of the chosen search algorithm are "good" to solve a particular class of problems.
- Write a program that selects "good parameter" settings based on problem characteristics (frequently very difficult) relying on machine learning

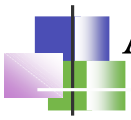
A* Search

- The greedy best-first search does not consider how costly it was to get to a node.
  - f(n) = h(n)
- Idea: avoid expanding paths that are already expensive
- Combine g(n), the cost to reach node n, with h(n)
  - f(n) = g(n) + h(n)
  - estimated cost of cheapest solution through n



A* Search

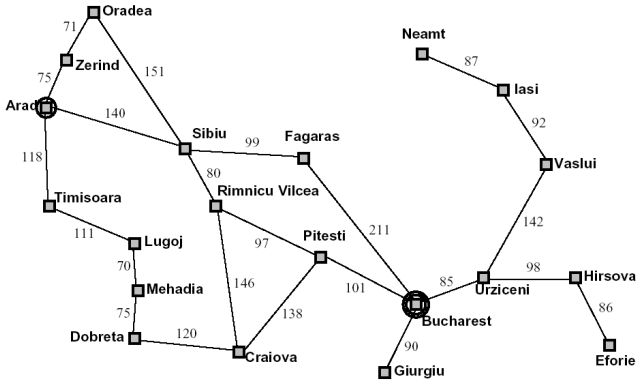
- When h(n) = actual cost to goal
  - Only nodes in the correct path are expanded
  - Optimal solution is found
- When h(n) < actual cost to goal
  - Additional nodes are expanded
  - Optimal solution is found
- When h(n) > actual cost to goal
  - Optimal solution can be overlooked



A* Search

- Complete
  - Yes, unless there are infinitely many nodes with f <= f(G)
- Time
  - Exponential in [relative error of h x length of soln]
  - The better the heuristic, the better the time
    - Best case h is perfect, O(d)
    - Worst case h = 0, $O(b^d)$ same as BFS
- Space
  - Keeps all nodes in memory and save in case of repetition
  - This is $O(b^d)$ or worse
  - A* usually runs out of space before it runs out of time
- Optimal
  - Yes, cannot expand f_{i+1} unless f_i is finished

Route Finding



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A* Example

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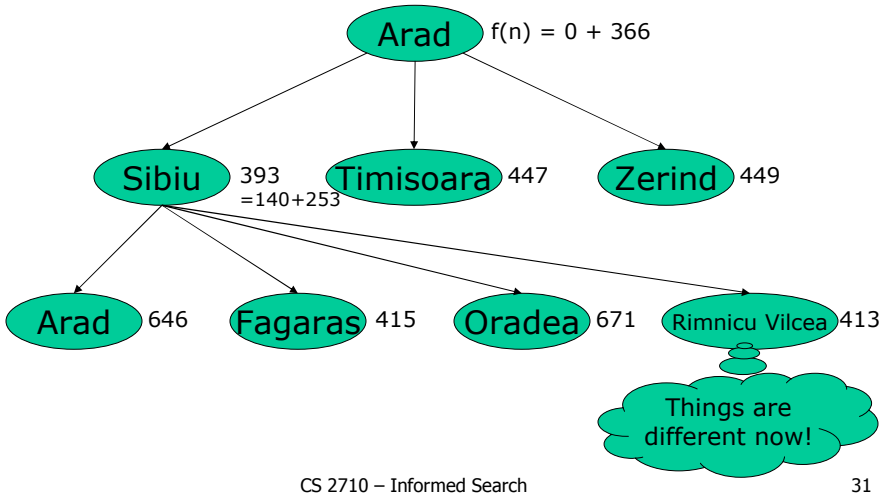
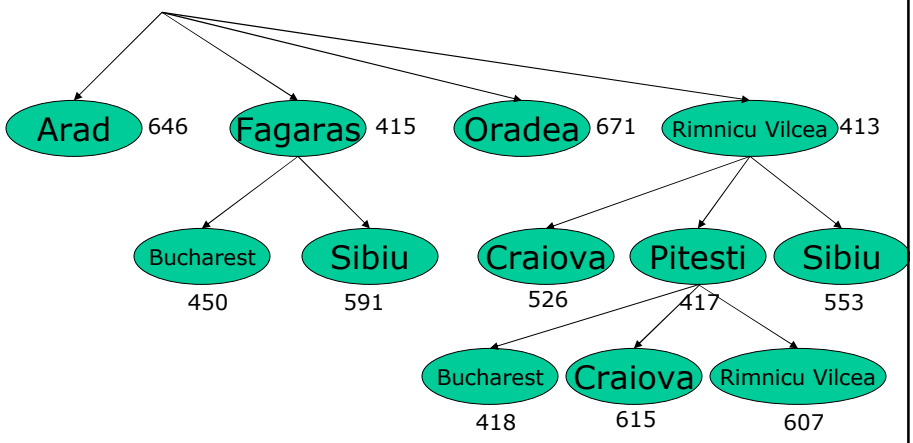
A* Search



A* Search Continued



A* Properties review

- Complete
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- Time
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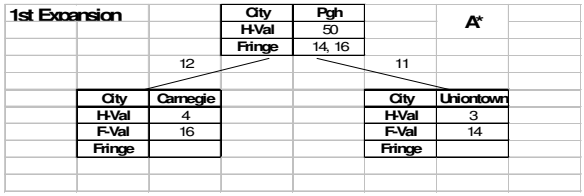
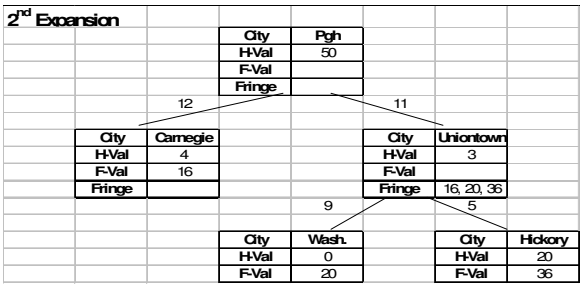
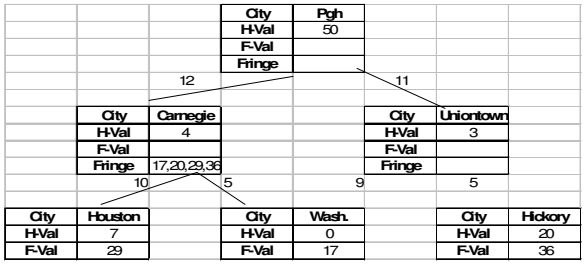
A* Exercise

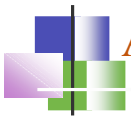


A* Exercise



A* Exercise





A* Search; complete

- A* is complete.

  A* builds search "bands" of increasing f(n)
  At all points f(n) < C*
  Eventually we reach the "goal contour"

- Optimally efficient
- Most times exponential growth occurs

Memory Bounded Heuristic Search

- Ways of getting around memory issues of A*:
- IDA* (iterative deepening algorithm)
  - Cutoff = f(n) instead of depth
- Recursive Best First Search
  - Mimic standard BFS, but use linear space!
  - Keeps track of best f(n) from alternate paths

RBFS

- F-limit: keeps track of the f-value of the best alternative path available
- F-value replacement: as the recursion unwinds, replaces f-value of each node with the best f-value of its children.

RBFS Exercise

1st Expansion

City	Pgh
H-Val	50
F-Limit	

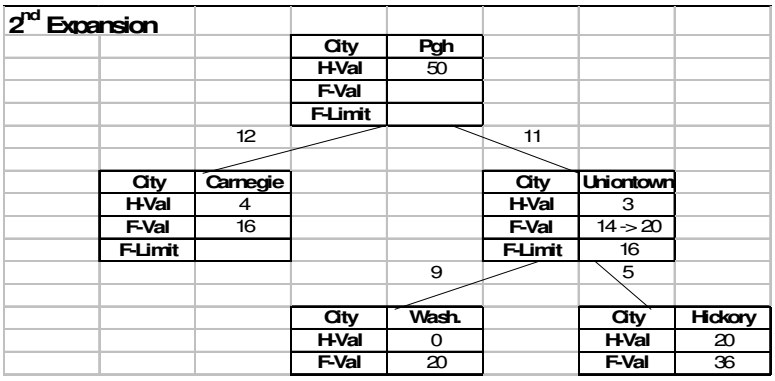
12 11

City	Carnegie
H-Val	4
F-Val	16
F-Limit	

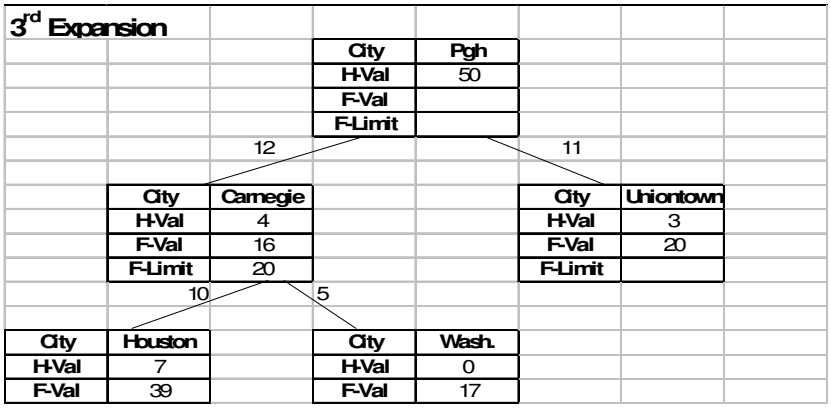
City	Uniontown
H-Val	3
F-Val	14
F-Limit	16

RBFS

RBFS Exercise



RBFS Exercise



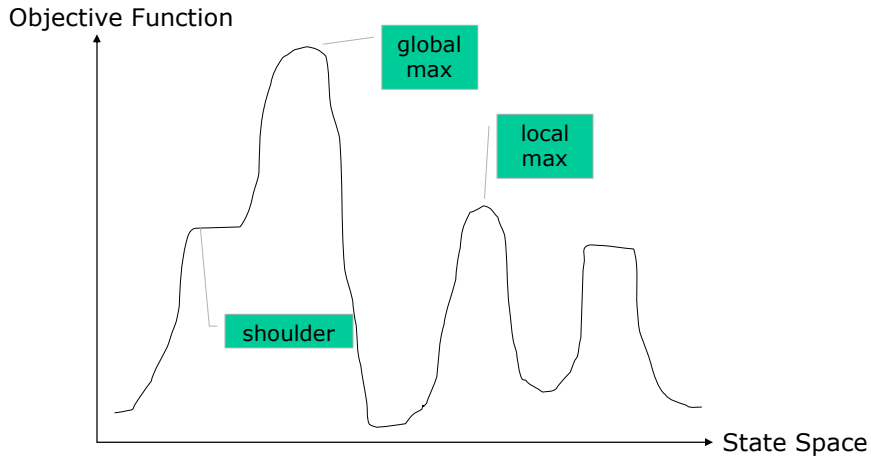
RBFS Review

- F-limit: keeps track of the f-value of the best alternative path available
- F-value replacement: as the recursion unwinds, replaces f-value of each node with the best f-value of its children.
- Disad's: excessive node regeneration from recursion
- Too little memory! → use memory-bounded approaches
  - Cutoff when memory bound is reached and other constraints

Local Search / Optimization

- Idea is to find the *best* state.
- We don't really care how to get to the best state, just that we get there.
- The best state is defined according to an *objective function*
  - Measures the "fitness" of a state.
- Problem: Find the optimal state
  - The one that maximizes (or minimizes) the objective function.

State Space Landscapes



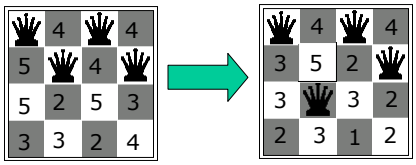
Problem Formulation

- Complete-state formulation
  - Start with an approximate solution and perturb
- n-queens problem
  - Place n queens on a board so that no queen is attacking another queen.

Problem Formulation

- Initial State: n queens placed randomly on the board, one per column.
- Successor function: States that obtained by moving one queen to a new location in its column.
- Heuristic/objective function: The number of pairs of attacking queens.

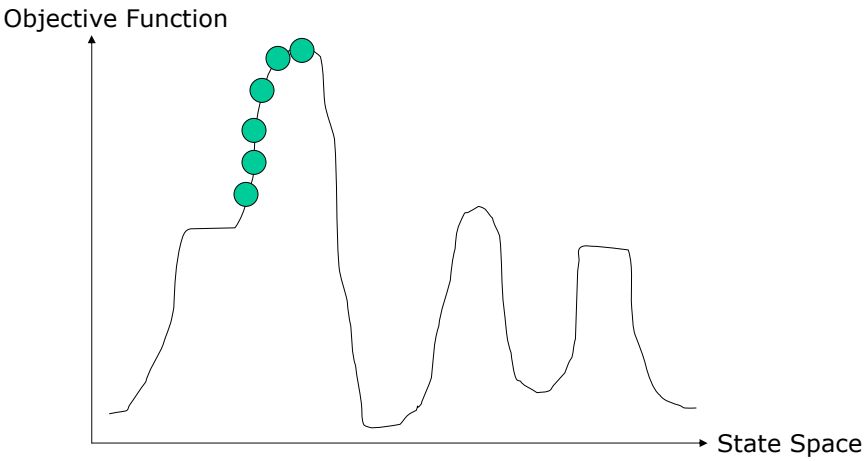
n-Queens



Local Search Algorithms

- Hill climbing
- Simulated annealing
- Local beam search
- Genetic Algorithms

Hill Climbing (or Descent)



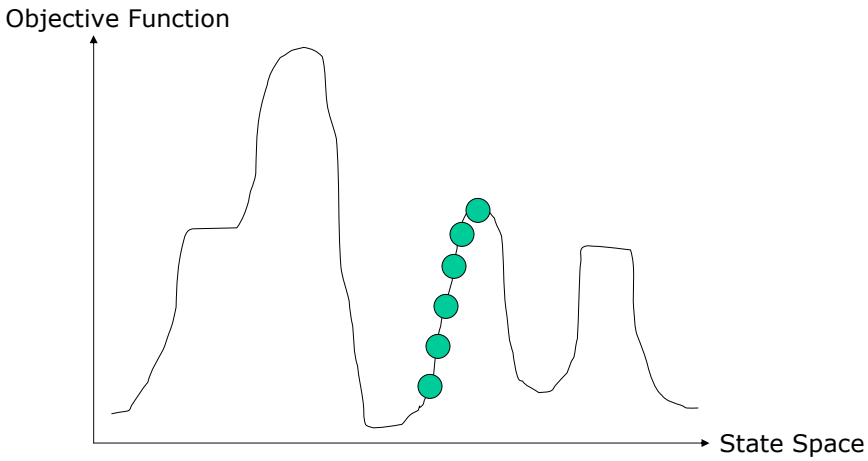
Hill Climbing Pseudo-code

- "Like climbing Everest in thick fog with amnesia"

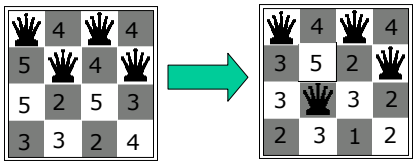
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
```


Hill Climbing Problems



n-Queens



What happens if we move 3rd queen?

Possible Improvements

- Stochastic hill climbing
  - Choose at random from uphill moves
  - Probability of move could be influenced by steepness
- First-choice hill climbing
  - Generate successors at random until one is better than current.
- Random-restart
  - Execute hill climbing several times, choose best result.
  - If p is probability of a search succeeding, then expected number of restarts is 1/p.

Simulated Annealing

- Similar to stochastic hill climbing
  - Moves are selected at random
  - If a move is an improvement, accept
  - Otherwise, accept with probability less than 1.
- Probability gets smaller as time passes and by the amount of "badness" of the move.

Simulated Annealing Algorithm

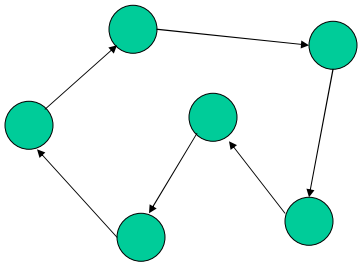
function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling the probability of downward steps

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow$ *schedule*[t]
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Success

Traveling Salesperson Problem

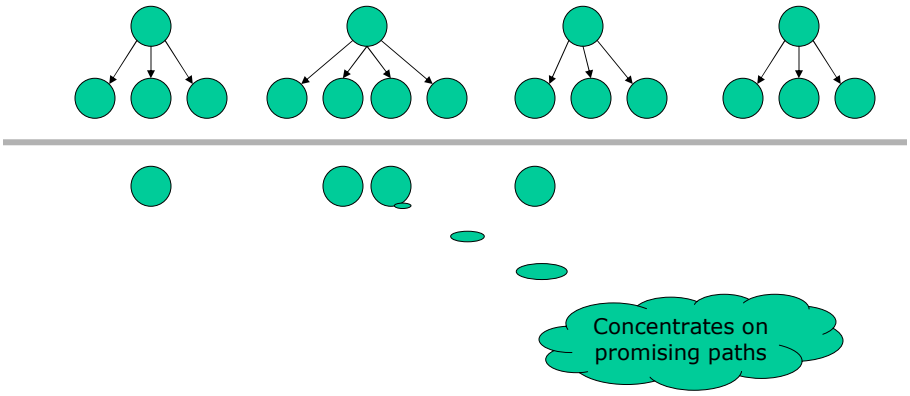
- Tour of cities
- Visit each one exactly once
- Minimize distance/cost/etc.



Local Beam Search

- Keep k states in memory instead of just one
- Generate successors of all k states
- If one is a goal, return the goal
- Otherwise, take k best successors and repeat.

Local Beam Search



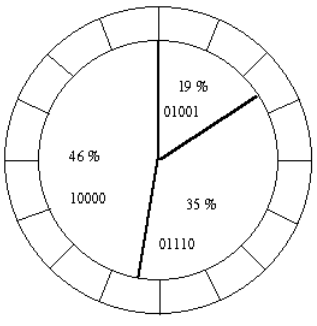
Local Beam Search

- Initial k states may not be diverse enough
  - Could have clustered around a local max.
- Improvement is *stochastic beam search*
  - Choose k states at random, with probability of choice an increasing function of its value.

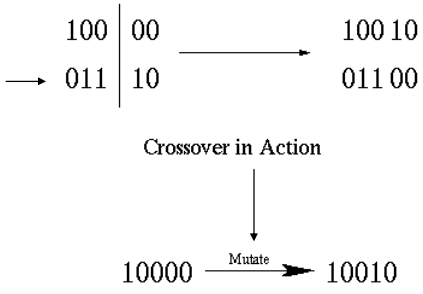
Genetic Algorithms

- Variant of stochastic beam search
- Successor states are generated by combining two parent states
  - Hopefully improves diversity
- Start with k states, the *population*
- Each state, or *individual*, represented as a string over a finite alphabet (e.g. DNA)
- Each state is rated by a *fitness function*
- Select parents for reproduction using the fitness function

Genetic Algorithms



Roulette Wheel Selection



Taken from http://www.cs.qub.ac.uk/~M.Sullivan/ga/ga_index.html

A Genetic Algorithm

```
function GENETIC-ALGORITHM( population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
      new_population ← empty set
      loop for i from 1 to SIZE( population) do
          x ← RANDOM-SELECTION( population, FITNESS-FN)
          y ← RANDOM-SELECTION( population, FITNESS-FN)
          child ← REPRODUCE(x, y)
          if (small random probability) then child ← MUTATE(child)
          add child to new_population
      population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN

function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```