Solving problems by searching

Chapter 3

Outline

- Problem-solving agents
- Problem formulation
- Example problems
- Basic search algorithms

Goal-based Agents

Agents that take actions in the pursuit of a goal or goals.

Goal-based Agents

- What should a goal-based agent do when none of the actions it can currently perform results in a goal state?
- Choose an action that at least leads to a state that is closer to a goal than the current one is.

Goal-based Agents

Making that work can be tricky:

- What if one or more of the choices you make turn out not to lead to a goal?
- What if you're concerned with the best way to achieve some goal?
- What if you're under some kind of resource constraint?

Problem Solving as Search

One way to address these issues is to view goal-attainment as problem solving, and viewing that as a search through a state space.

In chess, e.g., a state is a board configuration

Problem-solving agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
       goal ← FORMULATE-GOAL(state)
       problem ← FORMULATE-PROBLEM(state, goal)
       seq ← SEARCH( problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```


Problem Solving

A problem is characterized as:

- An initial state
- A set of actions
- A goal test
- A cost function

Problem Solving

A problem is characterized as:

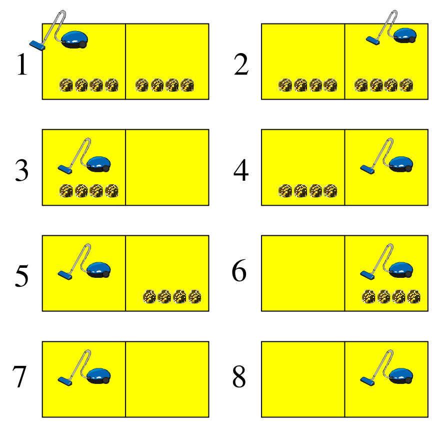
- An initial state
- A set of actions
  - successors: state → set of states
- A goal test
  - goalp: state → true or false
- A cost function
  - edgecost: edge between states → cost

Example Problems

- Toy problems (but sometimes useful)
  - Illustrate or exercise various problem-solving methods
  - Concise, exact description
  - Can be used to compare performance
  - *Examples*: 8-puzzle, 8-queens problem, Cryptarithmetic, Vacuum world, Missionaries and cannibals, simple route finding
- Real-world problem
  - More difficult
  - No single, agreed-upon description
  - *Examples*: Route finding, Touring and traveling salesperson problems, VLSI layout, Robot navigation, Assembly sequencing

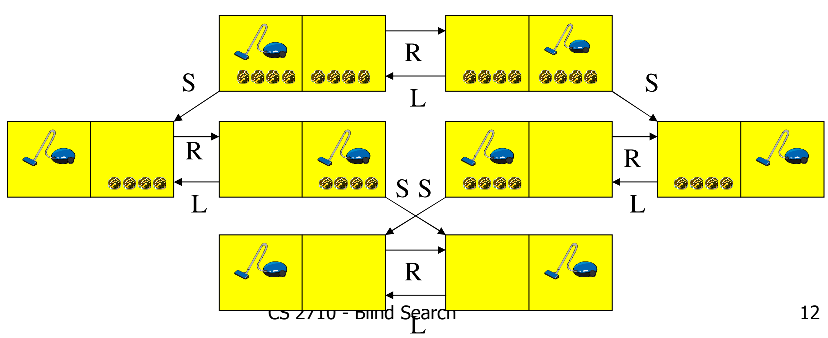
Toy Problems: *The vacuum world*

- The vacuum world
  - The world has only two *locations*
  - Each location may or may not contain *dirt*
  - The agent may be in one location or the other
  - 8 possible *world states*
  - Three possible actions: *Left, Right, Suck*
  - *Goal*: clean up all the dirt



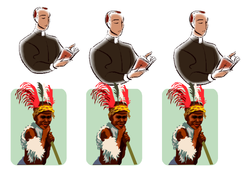
Toy Problems: *The vacuum world*

- *States*: one of the 8 states given earlier
- *Actions*: move left, move right, suck
- *Goal test*: no dirt left in any square
- *Path cost*: each action costs one



Missionaries and cannibals

- Missionaries and cannibals
  - Three missionaries and three cannibals want to cross a river
  - There is a boat that can hold two people
  - Cross the river, but make sure that the missionaries are not outnumbered by the cannibals on either bank
- Needs a lot of *abstraction*
  - Crocodiles in the river, the weather and so on
  - Only the endpoints of the crossing are important
  - Only two types of people



Missionaries and cannibals

- Problem formulation
  - *States*: ordered sequence of three numbers representing the number of missionaries, cannibals and boats on the bank of the river from which they started. The start state is (3, 3, 1)
  - *Actions*: take two missionaries, two cannibals, or one of each across in the boat
  - *Goal test*: reached state (0, 0, 0)
  - *Path cost*: number of crossings

Real-world problems

- Route finding
  - Specified locations and transition along links between them
  - *Applications*: routing in computer networks, automated travel advisory systems, airline travel planning systems
- Touring and traveling salesperson problems
  - "Visit every city on the map at least once and end in Bucharest"
  - Needs information about the visited cities
  - *Goal*: Find the shortest tour that visits all cities
  - *NP-hard*, but a lot of effort has been spent on improving the capabilities of TSP algorithms
  - *Applications*: planning movements of automatic circuit board drills

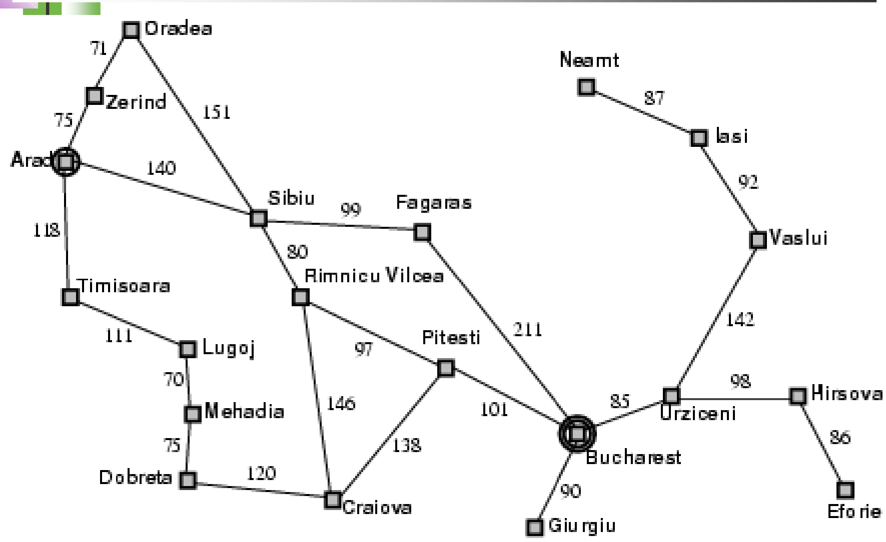
What is a Solution?

- A sequence of actions that when performed will transform the initial state into a goal state (e.g., the sequence of actions that gets the missionaries safely across the river)
- Or sometimes just the goal state (e.g., infer molecular structure from mass spectrographic data)

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

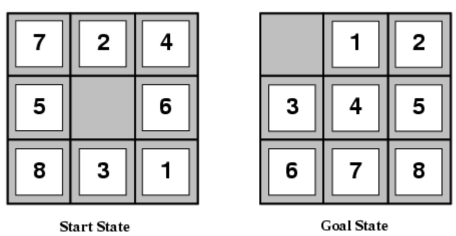
Example: Romania



Selecting a state space

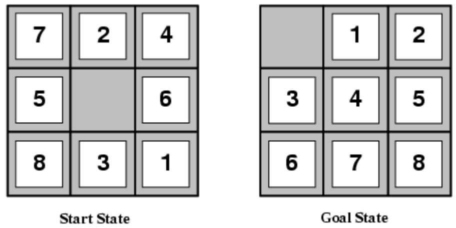
- Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad‟ must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Example: The 8-puzzle



- states?
- actions?
- goal test?
- path cost?

Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

Initial Assumptions

- The agent knows its current state
- Only the actions of the agent will change the world
- The effects of the agent's actions are known and deterministic

All of these are defeasible… likely to be wrong in real settings.

Another Assumption

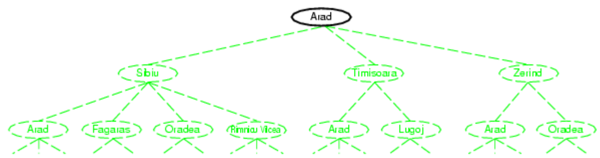
- Searching/problem-solving and acting are distinct activities
- First you search for a solution (in your head) then you execute it

Tree search algorithms

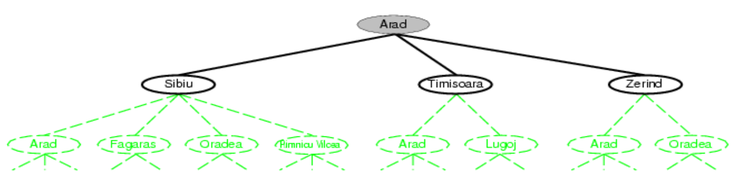
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
```

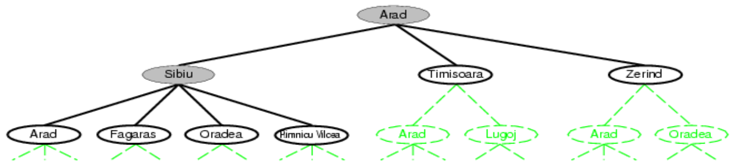
Tree search example



Tree search example



Tree search example



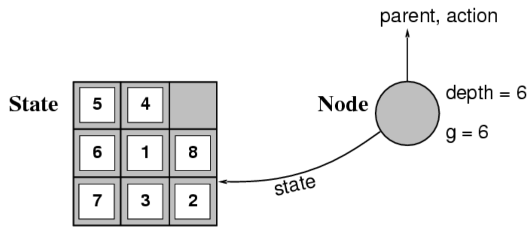
Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
      fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
      s ← a new NODE
      PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
      PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
      DEPTH[s] ← DEPTH[node] + 1
      add s to successors
   return successors
```

Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.

Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

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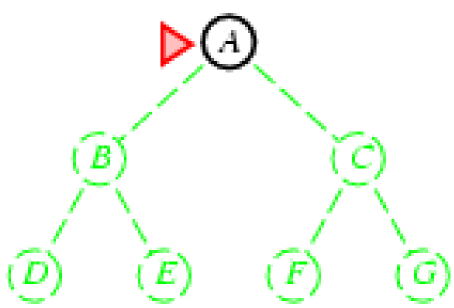
Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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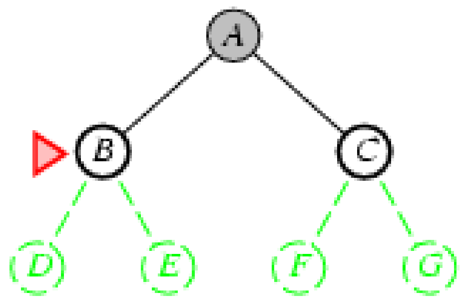

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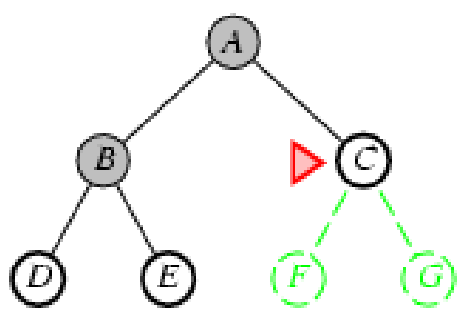

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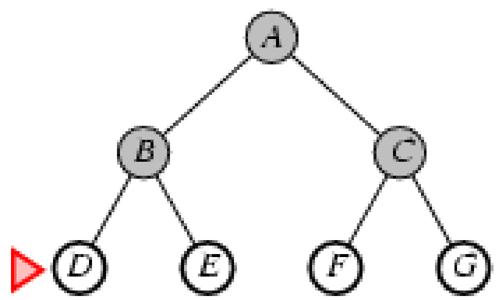


Properties of breadth-first search

- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)

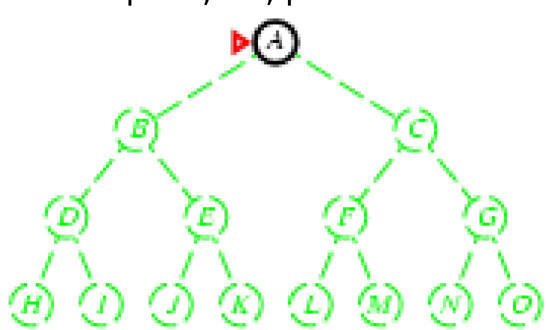
- Space is the bigger problem (more than time)

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with $g \le$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with $g \le$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of $g(n)$

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



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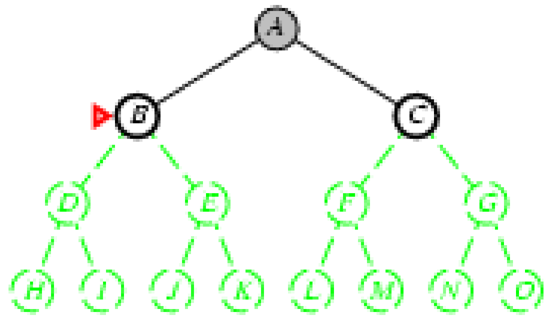

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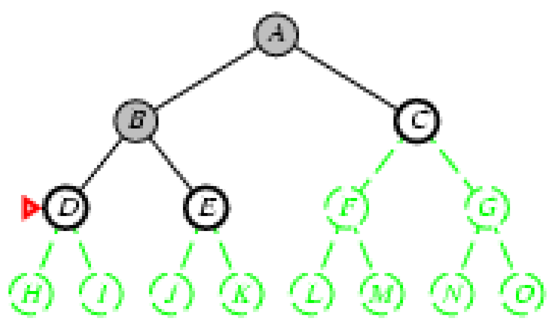

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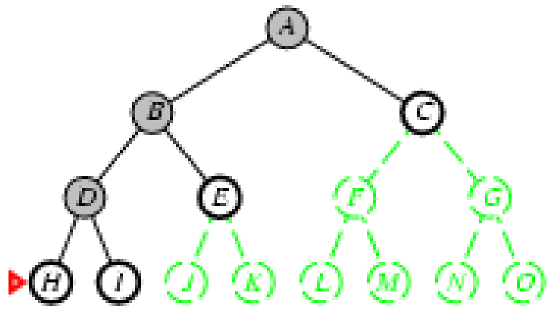

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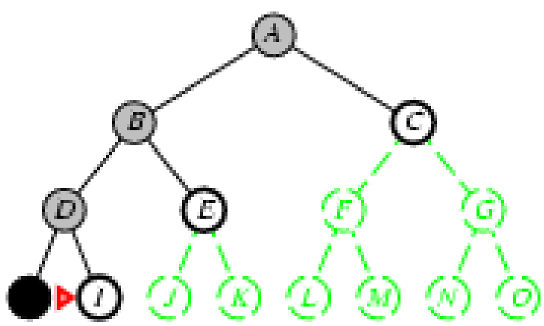

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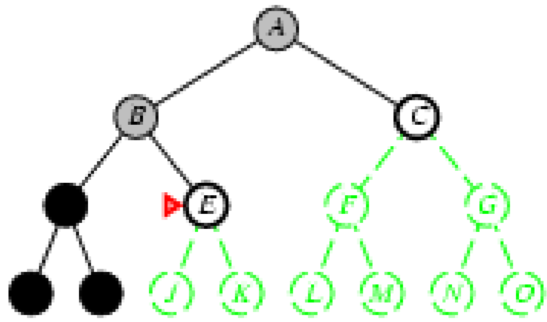

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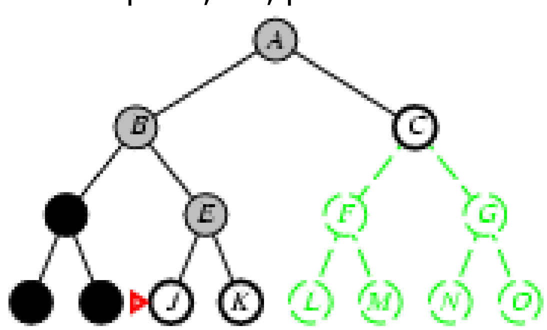

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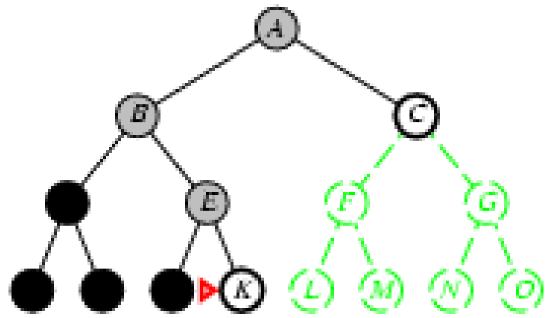

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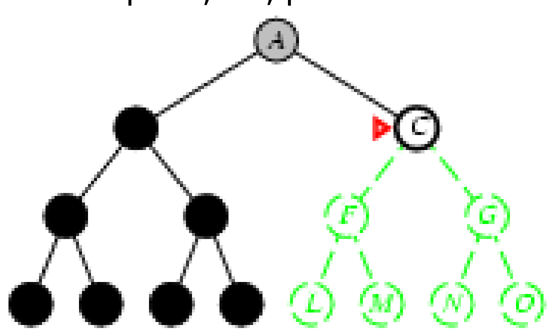

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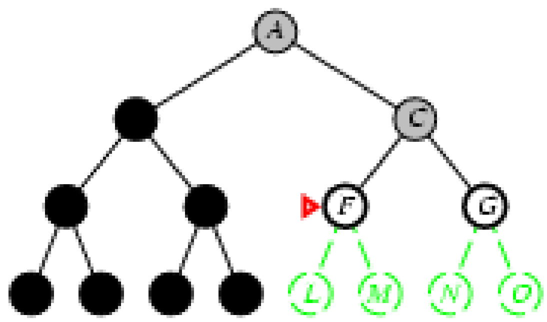

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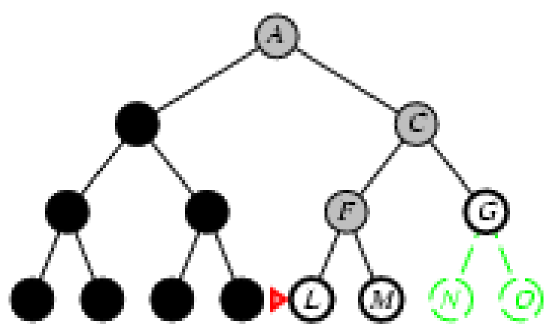

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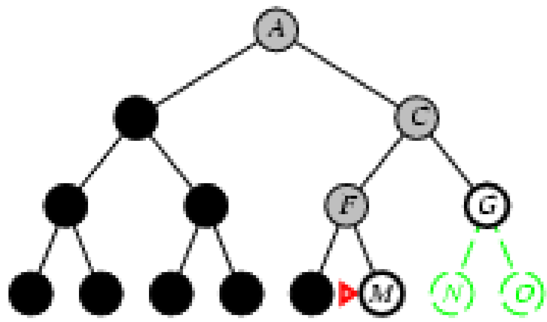


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    → complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

- Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
      result ← RECURSIVE-DLS(successor, problem, limit)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```


Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
   inputs: problem, a problem

   for depth ← 0 to ∞ do
      result ← DEPTH-LIMITED-SEARCH( problem, depth)
      if result ≠ cutoff then return result
```

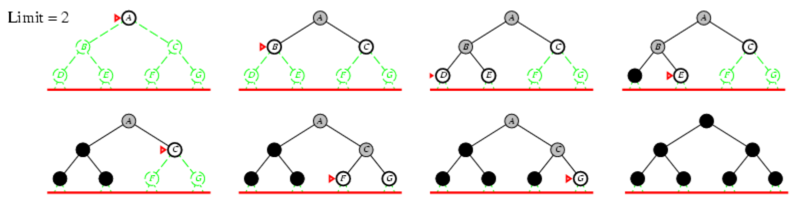
Iterative deepening search l =0

Limit = 0

Iterative deepening search l =1

Limit = 1

Iterative deepening search l =2



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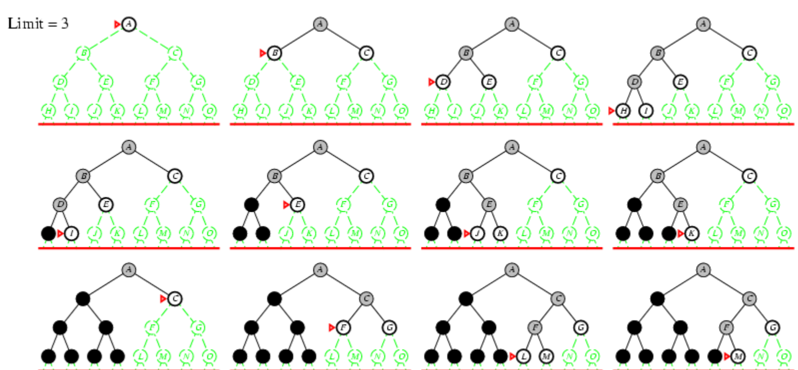
Iterative deepening search l =3



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Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\ b\verb|^|1 + (d-1)b\verb|^|2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
  - N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
  - N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456

- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\ b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms