7.12

a. $P \Rightarrow Q$ is equivalent to $\neg P \vee Q$ by implication elimination (Figure 7.11), and $\neg(P_1 \wedge \cdots \wedge P_m)$ is equivalent to $(\neg P_1 \vee \cdots \vee \neg P_m)$ by de Morgan's rule, so $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$ is equivalent to $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$.

8.7 The key idea is to see that the word "same" is referring to every *pair* of Germans. There are several logically equivalent forms for this sentence. The simplest is the Horn clause:

$$\forall x, y, l \;\; German(x) \wedge German(y) \wedge Speaks(x, l) \;\Rightarrow\; Speaks(y, l) \,.$$

8.8 $\forall x, y \;\; Spouse(x, y) \wedge Male(x) \;\Rightarrow\; Female(y)$. This axiom is no longer true in certain states and countries.

8.15 There are several problems with the proposed definition. It allows one to prove, say, $Adjacent([1,1],[1,2])$ but not $Adjacent([1,2],[1,1])$; so we need an additional symmetry axiom. It does not allow one to prove that $Adjacent([1,1],[1,3])$ is false, so it needs to be written as

$$\forall s_1, s_2 \;\; \Leftrightarrow \;\ldots$$

Finally, it does not work as the boundaries of the world, so some extra conditions must be added.

8.16 We need the following sentences:

$$\forall s_1 \;\; Smelly(s_1) \;\Leftrightarrow\; \exists s_2 \;\; Adjacent(s_1, s_2) \wedge In(Wumpus, s_2)$$
$$\exists s_1 \;\; In(Wumpus, s_1) \wedge \forall s_2 \;\; (s_1 \neq s_2) \;\Rightarrow\; \neg In(Wumpus, s_2) \,.$$

9.4 This is an easy exercise to check that the student understands unification.

a. $\{x/A, y/B, z/B\}$ (or some permutation of this).

b. No unifier (x cannot bind to both A and B).

c. $\{y/John, x/John\}$.

d. No unifier (because the occurs-check prevents unification of y with $Father(y)$).

9.18 This is a form of inference used to show that Aristotle's syllogisms could not capture all sound inferences.

a. $\forall x \;\; Horse(x) \;\Rightarrow\; Animal(x)$
$\forall x, h \;\; Horse(x) \wedge HeadOf(h, x) \;\Rightarrow\; \exists y \;\; Animal(y) \wedge HeadOf(h, y)$

b. $A.\; \neg Horse(x) \vee Animal(x)$
$B.\; Horse(G)$
$C.\; HeadOf(H, G)$
$D.\; \neg Animal(y) \vee \neg HeadOf(H, y)$
(Here $A.$ comes from the first sentence in **a.** while the others come from the second. H and G are Skolem constants.)

c. Resolve D and C to yield $\neg Animal(G)$. Resolve this with A to give $\neg Horse(G)$. Resolve this with B to obtain a contradiction.