

UNCERTAINTY: WUMPUS WORLD

CHAPTER 13

Wumpus World

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

$P_{ij} = true$ iff $[i, j]$ contains a pit

$B_{ij} = true$ iff $[i, j]$ is breezy

Include only $B_{1,1}, B_{1,2}, B_{2,1}$ in the probability model

Specifying the probability model

The full joint distribution is $\mathbf{P}(P_{1,1}, \dots, P_{4,4}, B_{1,1}, B_{1,2}, B_{2,1})$

Apply product rule: $\mathbf{P}(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,1}, \dots, P_{4,4})\mathbf{P}(P_{1,1}, \dots, P_{4,4})$

(Do it this way to get $P(\textit{Effect} \mid \textit{Cause})$.)

First term: 1 if pits are adjacent to breezes, 0 otherwise

Second term: pits are placed randomly, probability 0.2 per square:

$$\mathbf{P}(P_{1,1}, \dots, P_{4,4}) = \prod_{i,j=1,1}^{4,4} \mathbf{P}(P_{i,j}) = 0.2^n \times 0.8^{16-n}$$

for n pits.

Observations and query

We know the following facts:

$$b = \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$$

$$known = \neg p_{1,1} \wedge \neg p_{1,2} \wedge \neg p_{2,1}$$

Query is $\mathbf{P}(P_{1,3} | known, b)$

Define $Unknown = P_{ij}$ s other than $P_{1,3}$ and $Known$

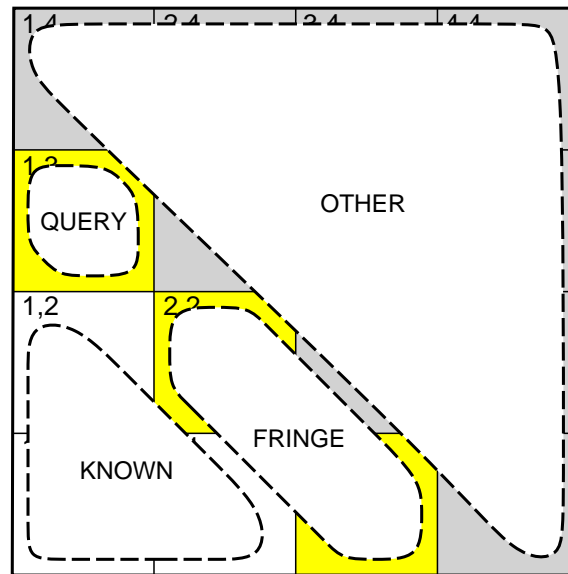
For inference by enumeration, we have

$$\mathbf{P}(P_{1,3} | known, b) = \alpha \sum_{unknown} \mathbf{P}(P_{1,3}, unknown, known, b)$$

Grows exponentially with number of squares!

Using conditional independence

Basic insight: observations are conditionally independent of other hidden squares given neighbouring hidden squares



Define $Unknown = Fringe \cup Other$

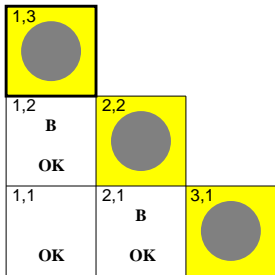
$$\mathbf{P}(b|P_{1,3}, Known, Unknown) = \mathbf{P}(b|P_{1,3}, Known, Fringe)$$

Manipulate query into a form where we can use this!

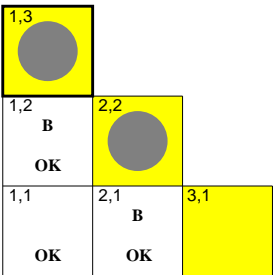
Using conditional independence contd.

$$\begin{aligned}
 \mathbf{P}(P_{1,3} | \textit{known}, b) &= \alpha \sum_{\textit{unknown}} \mathbf{P}(P_{1,3}, \textit{unknown}, \textit{known}, b) \\
 &= \alpha \sum_{\textit{unknown}} \mathbf{P}(b | P_{1,3}, \textit{known}, \textit{unknown}) \mathbf{P}(P_{1,3}, \textit{known}, \textit{unknown}) \\
 &= \alpha \sum_{\textit{fringe}} \sum_{\textit{other}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}, \textit{other}) \mathbf{P}(P_{1,3}, \textit{known}, \textit{fringe}, \textit{other}) \\
 &= \alpha \sum_{\textit{fringe}} \sum_{\textit{other}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}) \mathbf{P}(P_{1,3}, \textit{known}, \textit{fringe}, \textit{other}) \\
 &= \alpha \sum_{\textit{fringe}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}) \sum_{\textit{other}} \mathbf{P}(P_{1,3}, \textit{known}, \textit{fringe}, \textit{other}) \\
 &= \alpha \sum_{\textit{fringe}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}) \sum_{\textit{other}} \mathbf{P}(P_{1,3}) P(\textit{known}) P(\textit{fringe}) P(\textit{other}) \\
 &= \alpha P(\textit{known}) \mathbf{P}(P_{1,3}) \sum_{\textit{fringe}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}) P(\textit{fringe}) \sum_{\textit{other}} P(\textit{other}) \\
 &= \alpha' \mathbf{P}(P_{1,3}) \sum_{\textit{fringe}} \mathbf{P}(b | \textit{known}, P_{1,3}, \textit{fringe}) P(\textit{fringe})
 \end{aligned}$$

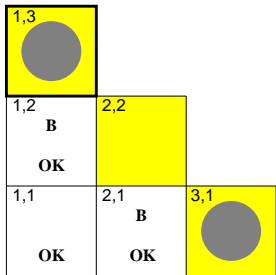
Using conditional independence contd.



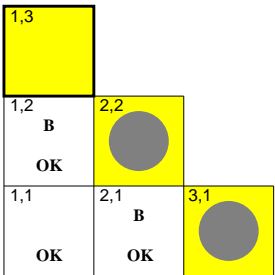
$$0.2 \times 0.2 = 0.04$$



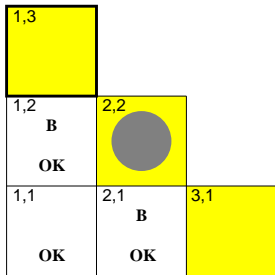
$$0.2 \times 0.8 = 0.16$$



$$0.8 \times 0.2 = 0.16$$



$$0.2 \times 0.2 = 0.04$$



$$0.2 \times 0.8 = 0.16$$

$$\mathbf{P}(P_{1,3} | \text{known}, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16) \rangle \approx \langle 0.31, 0.69 \rangle$$

$$\mathbf{P}(P_{2,2} | \text{known}, b) \approx \langle 0.86, 0.14 \rangle$$