CS2710/ISSP2160
Homework 4 Answer key
(Paper Problems)

Problem 2: Probability (10pts)

Let D denote the event "having the disease" and let + denote the event "test positive"

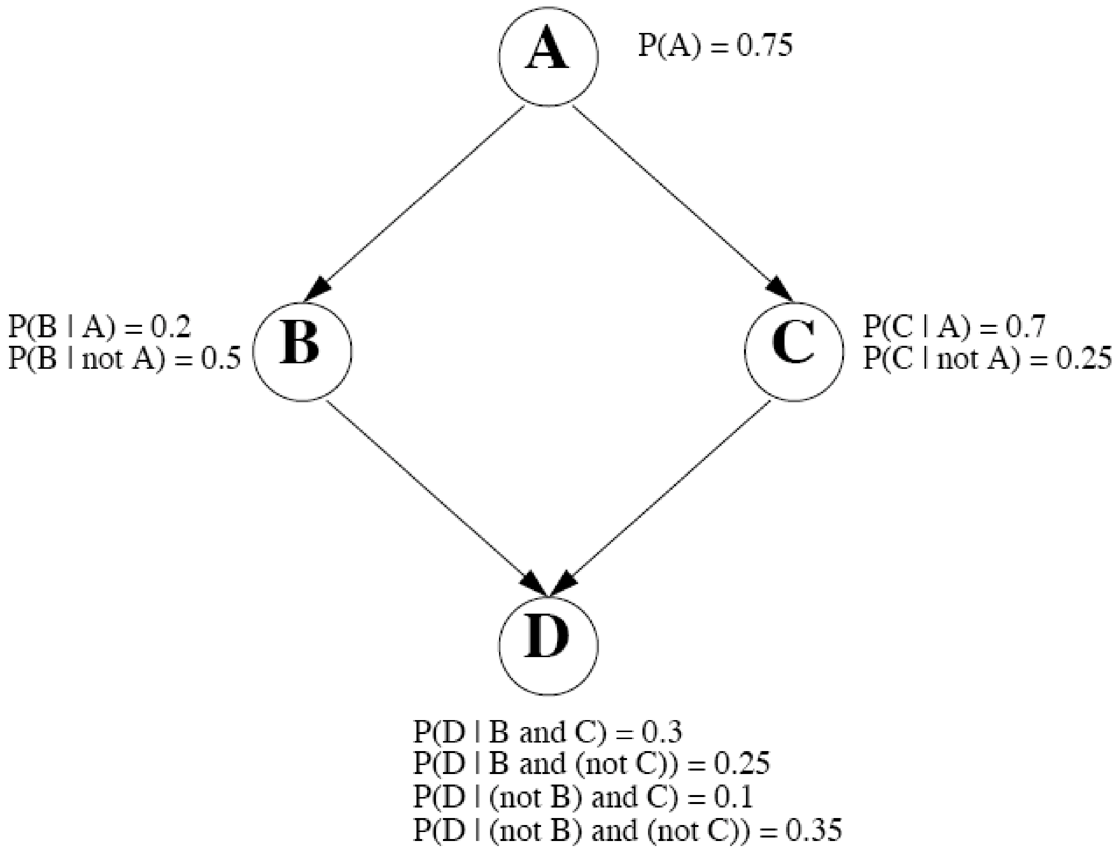
We are given the following information:
P(D) = 0.02
which implies P(not D) = 0.98
P(not + | D) = 0.06
which implies P(+ | D) = 0.94
P(+ | not D) = 0.09
First, we compute P(+)
= P(+ AND D) + P(+ AND (not D))
= P(+ | D) P(D) + P(+ | not D) P(not D)
= 0.94 x 0.02 + 0.09 x 0.98
= 0.107

We would like to know P(D | +)
= P(+ | D) x P(D) / P(+)
= 0.94 x 0.02 / 0.107
~= 0.1757

Problem 3: Probability (10pts)

a. P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2
b. The vector of probability values for the random variable Cavity has two values, which are listed in the order <true, false>. The probability of 'true' is 0.108+0.012+0.072+0.008=0.2. Then, we have **P**(Cavity) = <0.2, 0.8>
(note that bold-faced **P** refers to the probability-vector representation of the distribution of Cavity)
c. **P**(Toothache|cavity)=<(0.108+0.012)/0.2,(0.072+0.008)/0.2> = <0.6,0.4>
d. **P**(Cavity|toothache OR catch) =
<(0.108+0.012+0.072)/0.416,(0.016+0.064+0.144)/0.416> = <0.4615,0.5384>

Problem 4: Bayesian Networks (10pts)



$$P(D \mid A) = \sum_{(b,c)\in B\times C} P(D \mid (B,C) = (b,c)) \times P((B,C) = (b,c) \mid A)$$

$$= P(D \mid B \text{ and C}) \times P(B \text{ and C} \mid \text{A}) +$$

$$P(D \mid B \text{ and (not C)}) \times P(B \text{ and (not C)} \mid \text{A}) +$$

$$P(D \mid \text{(not B) and C}) \times P(\text{(not B) and C} \mid \text{A}) +$$

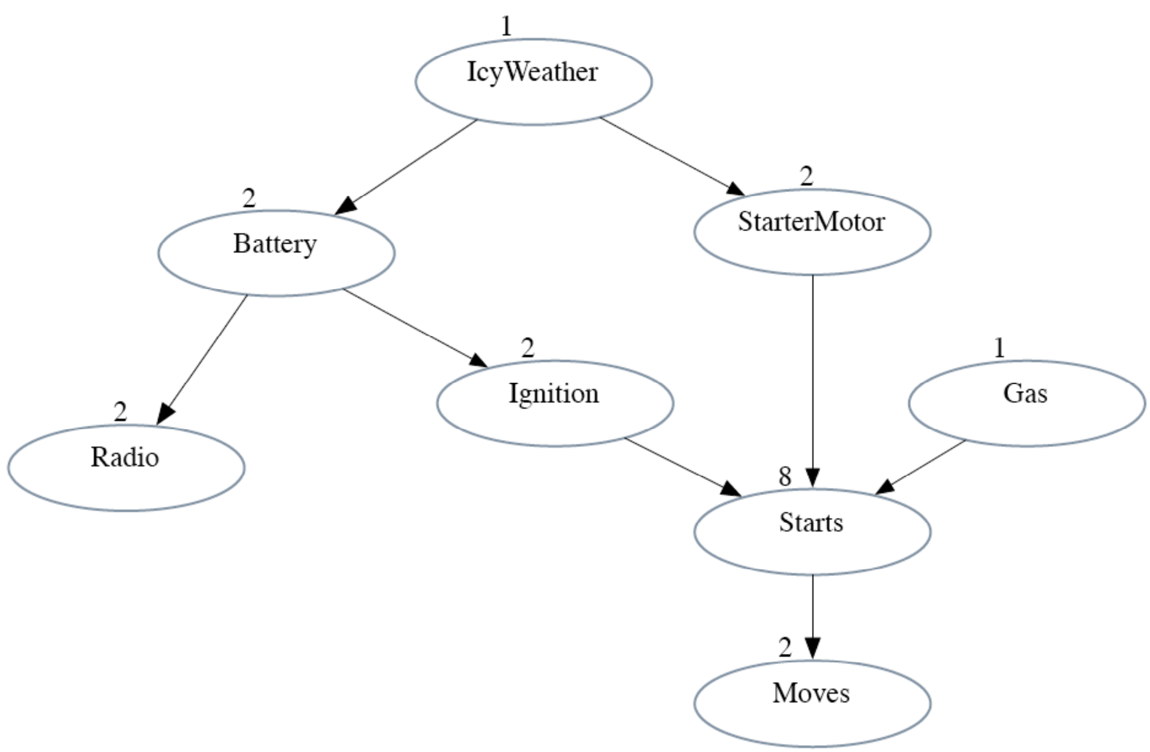
$$P(D \mid \text{(not B) and (not C)}) \times P(\text{(not B) and (not C)} \mid \text{A})$$

= (0.3 x 0.2 x 0.7) + (0.25 x 0.2 x 0.3) + (0.1 x 0.8 x 0.7) + (0.35 x 0.8 x 0.3)
= 0.042 + 0.015 + 0.056 + 0.084
= 0.197

Problem 5: Bayesian Networks (10 pts)

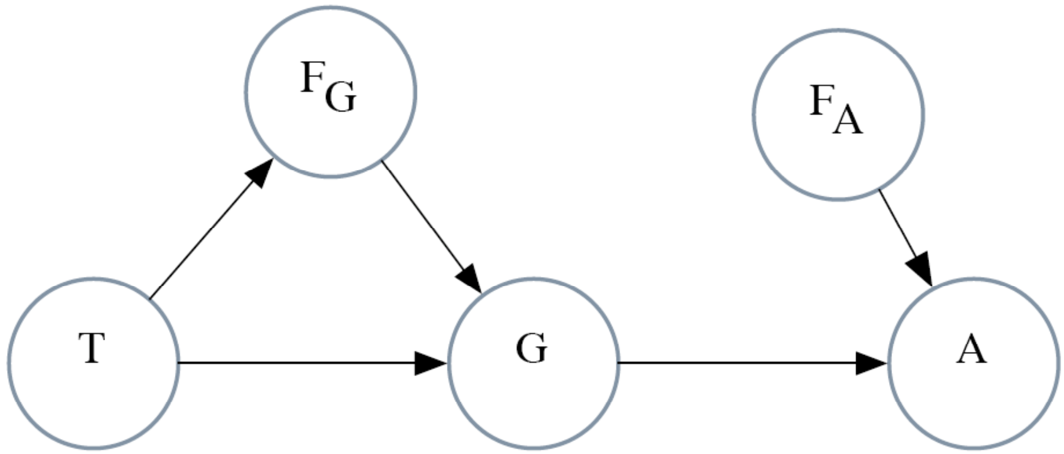
To add variables to an existing network, one may remove the existing arcs then identify which variables are direct causes or influences on what other ones, and build local parent/child graphs that way. Note that you should be careful to check for possible induced dependencies downstream.

a. IcyWeather is not caused by any car-related variables, so needs no parents. It directly affects the battery and the starter motor. StarterMotor is an additional precondition for Starts. The new network is shown below.
b. Resonable probabilities may vary a lot depending on the kind of car and perhaps the personal experience of the assessor.
c. With 8 boolean vars, the joint distribution has 2^8-1 = 255 independent entries.
d. Given the topology of the new network, the total number of independent CPT entries is 1+ 2 + 2 + 2 + 2 + 1 + 8 + 2 = 20



Problem 6: Bayesian Networks (10 pts)

a.



c.

	T=Normal		T=High	
	F_G	not F_G	F_G	not F_G
G = Normal	1-y	1-x	y	x
G = High	y	x	1-y	1-x

d.

	G= Normal		G=High	
	F_A	not F_A	F_A	not F_A
A	0	0	0	1
not A	1	1	1	0

Problem 7: Bayesian Networks (10 pts)

a. Diagram (i) is incorrect since it says that given the measurements M1 and M2, the number of stars is independent of focus. (ii) correctly represents the causal structure: each measurement is influenced by the actual number of stars and the focus, and the two telescopes are independent of each other. (iii) shows a correct but more complicated network—the one obtained by ordering the nodes M1, M2,N,F1,F2. If you order M2 before M1 you would get the same network except with the arrow from M1 to M2 reversed.

b. (ii) requires fewer parameters and is therefore better than (iii).