# CS 2710/ISSP 2160 

Homework 3 Answer key (Paper Problems)

## Problem 2: FOL Inference: Forward and Backward Chaining (15 pts)

(a) Knowledge base:

1. Programmer $(x) \wedge$ Emulator $(y) \wedge \operatorname{People}(z) \wedge \operatorname{Provide}(x, y, z) \Longrightarrow \operatorname{Criminal}(x)$
2. Use(Friends, $x) \wedge \operatorname{Runs}(x$, GameXgames $) \Longrightarrow$ Provide(SuperProgrammer, $x$, Friends)
3. Software $(x) \wedge \operatorname{Runs}(x$, GameXgames $) \Longrightarrow$ Emulator $(x)$
4. Programmer(SuperProgrammer)
5. People(Friends)
6. Software(Emulator1)
7. Use(Friends, Emulator1)
8. Runs(Emulator 1, GameX games)
(b) Forward chaining:

Step 1: Rules 2 and 3 can fire, but rule 2 fires first since it has a lower rule number. (Facts 8 and 7 match the premise of rule 2 with Emulator1 substituted for $x$ ). Firing rule 2 adds the fact Provide(SuperProgrammer, Emulator 1, Friends) to the knowledge base. Rule 2 will not fire again with these bindings.
Step 2: Rule 3 fires since facts 6 and 8 match its premise with Emulator1 substituted for $x$, adding the fact Emulator (Emulator 1 ) to the knowledge base.
Step 3: Rule 1 fires since facts 4 and 5 and the newly generated facts
Provide(SuperProgrammer, Emulator1, Friends) and Emulator(Emulator1) match this premise with SuperProgrammer substituted for $x$, Emulator 1 for $y$, and Friends for $z$, adding the fact Criminal(SuperProgrammer) to the knowledge base.
(c) Backward chaining:

We want to prove Criminal(SuperProgrammer). It is not in the knowledge base. It matches
the conclusion of only rule 1 with SuperProgrammer substituted for $x$. Thus, we need to prove the premise of rule 1, namely Programmer(SuperProgrammer), Emulator(y), People(z), and Provide(SuperProgrammer, $y, z$ ), for some objects $y$ and $z$. We consider these in turn.
Programmer (SuperProgrammer) is proven since it is in the knowledge base.
Emulator (y) does not match facts in the knowledge base but it matches the conclusion of rule 3 with $y$ equal to $x$. Thus, we need to prove the premise of rule 3, namely $\operatorname{Software}(y)$ and Runs(y,GameXgames). Software(y) is proven with Emulator1 substituted for $y$ since Software (Emulator 1 ) is in the knowledge base. We then need to prove
Runs(Emulator 1, GameXgames). This is proven since it is in the knowledge base as well.
People $(z)$ is proven with Friends substituted for $z$ since People(Friends) is in the knowledge base.

Finally, Provide(SuperProgrammer, y, z) now is Provide(SuperProgrammer, Emulator1, Friends), which is not in the knowledge base but matches the conclusion of rule 2 with Emulator1 substituted for $x$. Thus, we need to prove the premise of rule 2, namely Use(Friends, Emulator 1) and Runs(Emulator1, GameXgames), which are both in the knowledge base.
At this point, the proof of Criminal(SuperProgrammer) succeeds.

## Problem 3: FOL Inference: Resolution (15 pts)

The relevant definitions for the family are:

$$
\begin{array}{r}
\operatorname{Rel}(\text { Parent }, x, y) \wedge \operatorname{Male}(x) \Longleftrightarrow \operatorname{Rel}(\text { Father }, x, y) \\
\operatorname{Rel}(\text { Son }, x, y) \Longleftrightarrow \operatorname{Rel}(\text { Parent }, y, x) \wedge \operatorname{Male}(x) \\
\operatorname{Rel}(\text { Sibling }, x, y) \Longleftrightarrow \neg(x=y) \wedge \exists p \operatorname{Rel}(\operatorname{Parent}, p, x) \wedge \operatorname{Rel}(\text { Parent }, p, y) \\
\operatorname{Rel}\left(\text { Father }, x_{1}, y\right) \wedge \operatorname{Rel}\left(\text { Father }, x_{2}, y\right) \Longrightarrow x_{1}=x_{2} \tag{4}
\end{array}
$$

The facts of the case are:

$$
\begin{array}{r}
\neg \operatorname{Rel}(\text { Sibling, } M e, x) \\
\text { Male }(M r X) \\
\operatorname{Rel}(\text { Father, Father } X, M r X) \\
\operatorname{Rel}(\text { Father, FatherMe, Me) } \\
\operatorname{Rel}(\text { Son, Father } X, \text { FatherMe }) \tag{9}
\end{array}
$$

We want to be able to show that $M e$ us the only son of my father, and therefore that $M e$ is father of $M r X$, who is make, and therefore that "that man" is my son. The query we want us therefore:

$$
\begin{equation*}
\operatorname{Rel}(r, M r X, y) \tag{10}
\end{equation*}
$$

We want to be able to get back the answer $\{r / S o n, y / M e\}$. Translating 1-10 into CNF (and negating (10)) we get:

$$
\begin{aligned}
& \text { (1a) } \neg \operatorname{Rel}(\text { Parent }, x, y) \vee \neg \operatorname{Male}(x) \vee \operatorname{Rel}(\text { Father }, x, y) \\
& \text { (1b) } \neg \operatorname{Rel}(\text { Father }, x, y) \vee \operatorname{Male}(x) \\
& \text { (1c) } \neg \operatorname{Rel}(\text { Father }, x, y) \vee \operatorname{Rel}(\text { Parent }, x, y) \\
& \text { (2a) } \neg \operatorname{Rel}(\text { Son, } x, y) \vee \operatorname{Rel}(\text { Parent, } y, x) \\
& \text { (2b) } \neg \operatorname{Rel}(S o n, x, y) \vee \operatorname{Male}(x) \\
& \text { (2c) } \neg \operatorname{Rel}(\text { Parent, } y, x) \vee \neg \operatorname{Male}(x) \vee \operatorname{Rel}(S o n, x, y) \\
& \text { (3a) } \neg \operatorname{Rel}(\text { Sibling }, x, y) \vee \neg(x=y) \\
& \text { (3b) } \neg \operatorname{Rel}(\text { Sibling }, x, y) \vee \operatorname{Rel}(\text { Parent }, P(x, y), x) \\
& \text { (3c) } \neg \operatorname{Rel}(\text { Sibling }, x, y) \vee \operatorname{Rel}(\text { Parent }, P(x, y), y) \\
& (3 \mathrm{~d}) \neg \operatorname{Rel}(\text { Parent }, P(x, y), x) \vee \neg \operatorname{Rel}(\text { Parent }, P(x, y), y) \vee(x=y) \vee \operatorname{Rel}(\text { Sibling }, x, y) \\
& \text { (4) } \neg \operatorname{Rel}\left(\text { Father, } x_{1}, y\right) \vee \neg \operatorname{Rel}\left(\text { Father }, x_{2}, y\right) \vee\left(x_{1}=x_{2}\right) \\
& \text { (10) } \neg \operatorname{Rel}(r, M r X, y)
\end{aligned}
$$

The following lists the steps of the proof, with the resolvents of each step in braces on the left:
(11):\{1c,8\} Rel(Parent, FatherMe, Me)
(12):\{2a,9\} Rel(Parent,FatherMe, FatherX)
(13):\{3d,11\} $\neg \operatorname{Rel}($ Parent, Father $M e, y) \vee(M e=y) \vee \operatorname{Rel}($ Sibling $, M e, y)$
(14):\{5,13\} $\neg \operatorname{Rel}($ Parent, FatherMe, $y) \vee(M e=y)$
(15): $\{12,13\} \quad M e=$ Father $X \vee \operatorname{Rel}($ Sibling, Me, Father $X)$
(16): $\{5,15\} \quad M e=F a t h e r X$
(17):\{7,16\} Rel(Father, Me, $M r X)$
(18):\{1c,17\} Rel(Parent, Me, $M r X)$
(19):\{2c,18\} $\neg \operatorname{Male}(M r X) \vee \operatorname{Rel}(S o n, M r X, M e)$
(20):\{6,19\} $\operatorname{Rel}(S o n, M r X, M e)$
(21):\{10,20\} FALSE $\{r / S o n, y / M e\}$

## Problem 4: Planning: Partial Order (15 pts)

| Initial State: | B | C Goal: C | D | $\begin{gathered} \text { Operator } \\ \text { O1 } \\ \text { O2 } \\ \text { O3 } \\ \text { O4 } \end{gathered}$ | PreconditionAABF | $\begin{gathered} \text { Add } \\ \text { F } \\ \text { D } \\ \text { C } \\ \text { DEG } \end{gathered}$ | $\begin{gathered} \text { Delete } \\ \text { C } \\ \text { B } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

(a) Graphplan (Fig. 1)

- Two actions instances at level $i$ are mutex if either
- Inconsistent effects: the effect of one action is the negation of the other action effect, or
- Interference: one action deletes the precondition of the other, or
- Competing needs: the actions have preconditions that are mutually exclusive at level $i$ - 1
- Two propositions at level $i$ are mutex if all ways of achieving the propositions (i.e., actions at level $i-1$ ) are pairwise mutex.

Level 0 begins with the initial state of $A B C$
Level 1 consists of the possible actions that have preconditions satisfied from level 0 . All arcs drawn between levels indicate mutex relations. Action O1 and O3 are mutually exclusive (mutex) due to inconsistent effects. O1 and the maintenance action of C are also mutex for the same reason. O2 is mutex O3 by interference and mutex the maintenance action of $B$ by inconsistent effects.
Level 2 consists of the possible effects from the actions in level 1. A and B are possible due to the maintenance actions. C is possible from a maintenance action and action O3. D is possible from action O2, and F is possible from action O1. B D and C F are mutex by inconsistent support.
Level 3 contains all the actions and mutexes from level 1 with one additional action and two additional mutexes. Action O 4 is now possible due to the addition of F at level 2. Action O 4 is mutex the maintenance action of C by competing needs. Action O 3 is mutex the maintenance action of D by competing needs.
Level 4 consists of all the possible effects from the actions in level 3. A B C D F are possible from maintenance actions. C from action O3. D from action O2 or 03. F from action O1. E and G from action O4.
Graphplan proceeds by expanding the planning graph to level 2 . It then checks to see if effects present at this level can satisfy the goal requirements. In this case C and D are present but E is absent. Graphplan then proceed to expand the graph to level 4 . Level 4 contains effects that satisfy the goal conditions, so graphplan begins to search backward to find a valid plan. We discuss a successful backward search. An unsuccessful search would


Figure 1: Planning graph: The graph has been expanded two time steps which is sufficient to find a valid plan.
result in dead ends with no actions to choose within the mutex constraints which would then backtrack the search and continue. At level 3 graphplan chooses O3 to satisfy C and O4 to satisfy D and E , this is valid as there is no mutex between O 3 and O 4 . The preconditions for level 3, B and F, are now the new goals to achieve. At level 1 graphplan chooses the maintenance action of B to satisfy B and action O1 to satisfy F. The preconditions from level 1 are satisfied from the initial state so the plan is completed: O1 at time 1, O3 and O4 at time 2.
(b) Regression planner

The following diagrams (Fig. 2) will show one way that a partial order planner would find a solution the planning problem. It only displays a perfect selection sequence, but a real planner may choose incorrectly resulting in backtracking. Each square is an operator. The letters above the squares are preconditions and the letters below are effects. Single width arrow lines are ordering constraints, and double width arrow lines are causal links. Notice that not all ordering constraints are shown in the graph as causal links are assumed to be accompanied by a similar ordering constraint and ordering constraints that can be inferred from those shown are not displayed. For example every operator has an ordering constraint
of Start $\prec$ Operator $\prec$ Goal but these are not displayed if they can be inferred.
Step 0 The plan is created with only the start and goal states.
Step 1 The planner selects for a subgoal the precondition $D$ from the goal state. It then chooses the O4 operator to satisfy this subgoal. It places an ordering constrain Start $\prec \mathrm{O} 4 \prec$ Goal, a causal link $O 4 \xrightarrow{D}$ Goal and ordering constrain $O 4 \prec$ Goal (from now on we omit the mention of the ordering constraint that accompanies the causal link - it is now implied).

Step 2 The planner selects for a subgoal the precondition E from the goal state. It then chooses the O4 operator to satisfy this subgoal. It adds a causal link $O 4 \xrightarrow{E}$ Goal.
Step 3 The planner selects for a subgoal the precondition $F$ from operator O4. It then chooses the O1 operator to satisfy this subgoal. It adds an ordering constraint Start $\prec \mathrm{O} \prec \prec$ Goal and a causal link $O 1 \xrightarrow{F} O 4$.

Step 4 The planner selects for a subgoal the precondition A from operator O1. It then chooses the start state to satisfy this subgoal. It adds a causal link Start $\xrightarrow{A} O 1$.
Step 5 The planner selects for a subgoal the precondition $C$ from operator goal state. It then chooses the O3 operator to satisfy this subgoal. It places an ordering constrain Start $\prec$ $O 3 \prec$ Goal and a causal link $O 3 \xrightarrow{C}$ Goal.

Step 6 There is a threat between O3 and O1. The planner chooses to promote O1 by adding an ordering constraint $O 1 \prec O 3$.
Step 7 The planner selects for a subgoal the precondition B from operator O3. It then chooses the start state to satisfy this subgoal. It adds a causal link Start $\xrightarrow{B} O 3$.

Finished The planner has now reached a solution. O1 followed by O3 and O4 together or in any order.

## Problem 5: Planning: STRIPS (15 pts)

Actions: Action(Action : Go $(x, y), \operatorname{Precond}: A t(\operatorname{Shakey}, x) \wedge \operatorname{In}(x, r) \wedge \operatorname{In}(y, r), \operatorname{EFFECT}: \operatorname{At}(\operatorname{Shakey}, y) \wedge$ $\neg($ At $($ Shakey,$x)))$
Action(Action : Push $(b, x, y), \operatorname{Precond}: A t(S h a k e y, x) \wedge P u s h a b l e(b), \operatorname{EFFEct}: A t(b, y) \wedge$ At $($ Shakey,$y) \wedge \neg A t(b, x) \wedge \neg A t($ Shakey,$x))$
Action(Action : $\operatorname{ClimbUp}(b)$, Precond : At $($ Shakey,$x) \wedge A t(b, x) \wedge C l i m b a b l e(b)$, Effect : On(Shakey, b)^ On(Shakey, Floor))
Action(Action : ClimbDown(b), Precond : On(Shakey, b), Effect : On(Shakey, Floor)^ / On(Shakey, b))
Action(Action : TurnOn $(l)$, Precond : On $($ Shakey,$b) \wedge A t(S h a k e y, x) \wedge A t(l, x)$, Effect : TurnedOn(l))
Action(Action : TurnOff(l), Precond : On $($ Shakey, $b) \wedge A t(S h a k e y, x) \wedge A t(l, x)$, Effect : $\neg$ TurnedOn(l))


Figure 2: Steps taken by a partial order planner with perfect selection.

Initial State: $\operatorname{In}\left(\right.$ Switch $_{1}$, Room $\left._{1}\right) \wedge \operatorname{In}\left(\right.$ Door $_{1}$, Room $\left._{1}\right) \wedge \operatorname{In}\left(\right.$ Door $_{1}$, Corridor $)$
$\operatorname{In}\left(\right.$ Switch $_{1}$, Room $\left._{2}\right) \wedge \operatorname{In}\left(\right.$ Door $_{2}$, Room $\left._{2}\right) \wedge \operatorname{In}\left(\right.$ Door $_{2}$, Corridor $)$
$\operatorname{In}\left(S_{w i t c h}^{1}\right.$, Room $\left._{3}\right) \wedge \operatorname{In}\left(\right.$ Door $_{3}$, Room $\left._{3}\right) \wedge \operatorname{In}\left(\right.$ Door $_{3}$, Corridor $)$
$\operatorname{In}\left(\right.$ Switch $_{1}$, Room $\left._{4}\right) \wedge \operatorname{In}\left(\right.$ Door $_{4}$, Room $\left._{4}\right) \wedge \operatorname{In}\left(\right.$ Door $_{4}$, Corridor $)$
$\operatorname{In}\left(\right.$ Shakey, Room $\left._{3}\right) \wedge$ At $\left(\right.$ Shakey,$\left.X_{S}\right)$
$\operatorname{In}\left(\right.$ Box $_{1}$, Room $\left._{1}\right) \wedge \operatorname{In}\left(\right.$ Box $_{2}$, Room $\left._{1}\right) \wedge \operatorname{In}\left(\right.$ Box $_{3}$, Room $\left._{1}\right) \wedge \operatorname{In}\left(\right.$ Box $_{4}$, Room $\left._{1}\right)$
Climbable $\left(\right.$ Box $\left._{1}\right) \wedge$ Climbable $\left(\right.$ Box $\left._{2}\right) \wedge$ Climbable $\left(\right.$ Box $\left._{3}\right) \wedge$ Climbable $\left(\right.$ Box $\left._{4}\right)$
Pushable $\left(\right.$ Box $\left._{1}\right) \wedge$ Pushable $\left(\right.$ Box $\left._{2}\right) \wedge$ Pushable $\left(\right.$ Box $\left._{3}\right) \wedge$ Pushable $\left(\right.$ Box $\left._{4}\right)$
$A t\left(B o x_{1}, X_{1}\right) \wedge A t\left(B o x_{2}, X_{2}\right) \wedge A t\left(B o x_{3}, X_{3}\right) \wedge \operatorname{At}\left(B o x_{4}, X_{4}\right)$
TurnedOn $\left(\right.$ Switch $\left._{1}\right) \wedge$ TurnedOn $\left(\right.$ Switch $\left._{4}\right)$.
Plan: $G o\left(X_{S}\right.$, Door $\left._{3}\right)$
Go(Door ${ }_{3}$, Door $\left._{1}\right)$
Go( Door $\left._{1}, X_{2}\right)$
Push(Box ${ }_{2}, X_{2}$, Door $\left._{1}\right)$
Push(Box ${ }_{2}$, Door $_{1}$, Door $\left._{2}\right)$
Push(Box ${ }_{2}$, Door $_{2}$, Switch $\left._{2}\right)$

