Expectiminimax Algorithm

- EXPECTIMINIMAX gives perfect play for non-deterministic games
- Like MINIMAX, except add chance nodes
  - For max node return highest EXPECTIMINIMAX of SUCCESSORS
  - For min node return lowest EXPECTIMINIMAX of SUCCESSORS
  - For chance node return average of EXPECTIMINIMAX of SUCCESSORS
- Here exact values of evaluation function do matter
  ("probabilities", "expected gain", not just order)
- α-β pruning possible by taking weighted averages according to probabilities

*-Minimax

- α-β pruning possible by taking weighted averages according to probabilities
  - *-Minimax (B. Ballard, 1983)
  - 50% improvement with random node order
  - Order of magnitude improvement with optimal order
- Add *cut-offs to chance nodes*
  - Max and min nodes as in alpha-beta algorithm
  - Assume that all branches not searched have the worst-case result
  - Assume range of evaluating values is *bound by interval* $[L, U]$

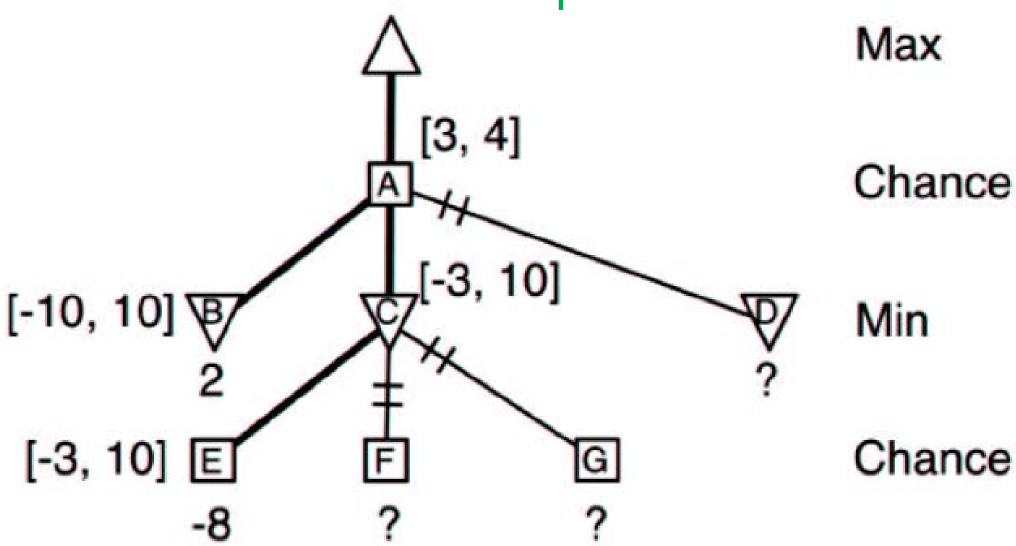
*-Minimax Cut-Off

- **Alpha cut-off** in chance node with N equally likely children

$$\frac{1}{N}\Big(\underbrace{(V_1 + \cdots + V_{l-1})}_{\text{explored}} + \underbrace{V_l}_{\text{current}} + \underbrace{U * (n - l)}_{\text{estimated future (worst case)}}\Big) \leq \alpha$$

- **Beta cut-off** in chance node with N equally likely children

$$\frac{1}{N}\Big(\underbrace{(V_1 + \cdots + V_{l-1})}_{\text{explored}} + \underbrace{V_l}_{\text{current}} + \underbrace{L * (n - l)}_{\text{estimated future (worst case)}}\Big) \geq \beta$$



- Bounds passed to **C** from **A** with $[L, U] = [-10, 10]$
  - $\frac{1}{3}(2 + V_l + L * 1) \geq \beta \Rightarrow V_l \geq 20$
  - $\frac{1}{3}(2 + V_l + U * 1) \leq \alpha \Rightarrow V_l \geq -3$