Adversarial Search

Chapter 5

Sections 1-5

Outline

- Games
- Optimal decisions
- α-β pruning
- Imperfect, real-time decisions
- Stochastic Games

Game Playing State-of-the-Art

- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Checkers is now solved!

- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue examined 200 million positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Othello:** Human champions refuse to compete against computers, which are too good.

- **Go:** Human champions are beginning to be challenged by machines, though the best humans still beat the best machines. In go, b > 300, so most programs use pattern knowledge bases to suggest plausible moves, along with aggressive pruning.

- **Pacman:** unknown

- AAAI conferences now have *general* game-playing competitions, with a $10K prize!

Game Search

- Game-playing programs developed by AI researchers since the beginning of the modern AI era (chess, checkers in 1950s)

- **Game Search**
  - Sequences of player's decisions *we control*
  - Decision of other player(s) *we do not control*

- **Contingency problem**: many possible opponent's moves must be "covered" by the solution
  - Introduces uncertainty to the game since we do not know what the opponent will do

- **Rational opponent:** maximizes it's own *utility* function

Types of Game Problems

- Adversarial
  - Win of one player is a loss of the other
  - Focus of this course
- Cooperative
  - Players have common interests and utility function
- A spectrum of others in between

Typical AI “Games”:

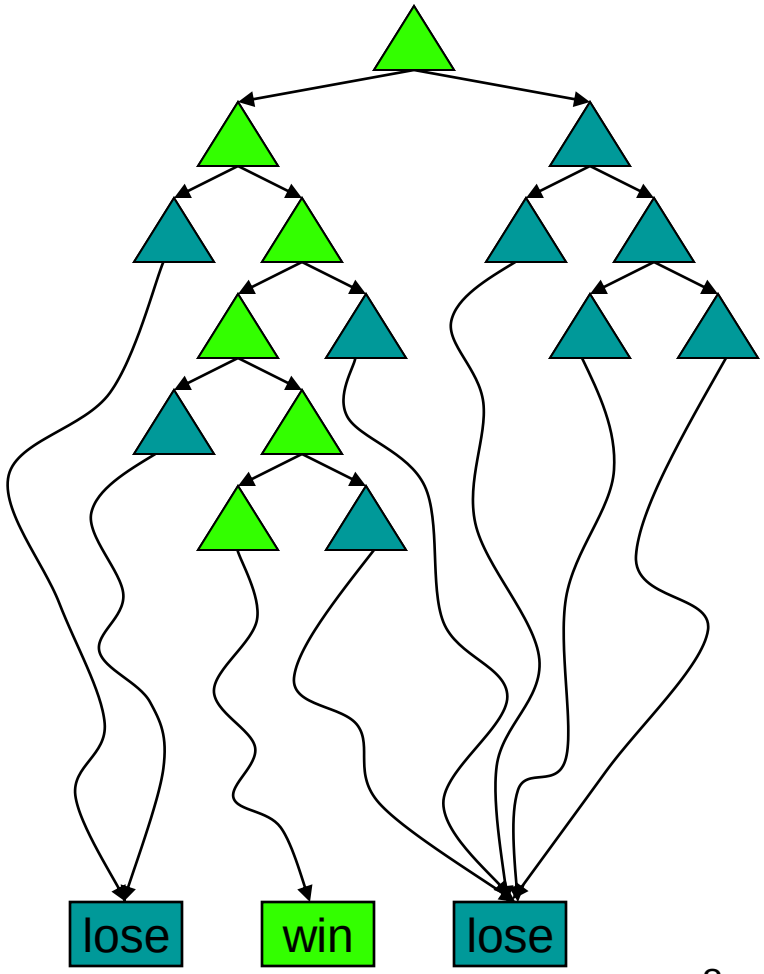
- Deterministic and Fully Observable Environment
- Two agents with turn-taking for actions
- Zero-sum (adverserial)
- Abstract (robotic soccer notable exception)
  - state easy to represent, few action choices, well-defined goals
  - hard to solve

Types of Games

	Deterministic	Chance
Perfect Information	Tic Tac Toe, Chess	Backgammon
Imperfect information	Stratego	Poker, Bridge

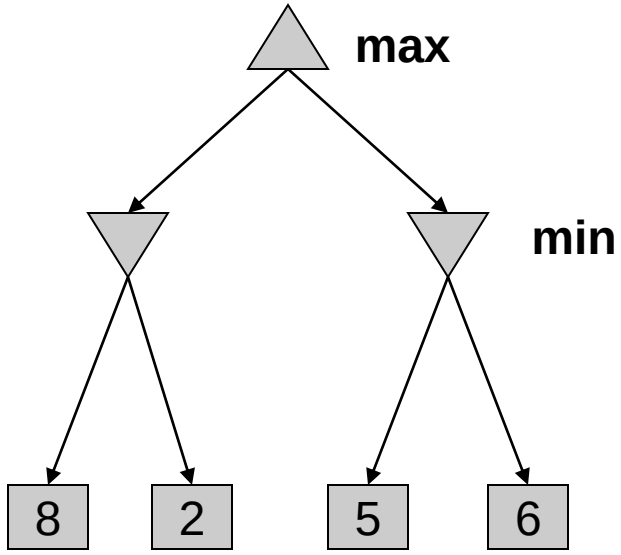
Deterministic Single-Player

- Deterministic, single player, perfect information:
  - Know the rules
  - Know what actions do
  - Know when you win
  - E.g. Freecell, 8-Puzzle, Rubik's cube
- … it's just search!
- Slight reinterpretation:
  - Each node stores a value: the best outcome it can reach
  - This is the maximal outcome of its children (the max value)
  - Note that we don't have path sums as before (utilities at end)
- After search, can pick move that leads to best node



Deterministic Two-Player

- E.g. tic-tac-toe, chess, checkers
- Zero-sum games
  - One player maximizes result
  - The other minimizes result
- **Minimax search**
  - A state-space search tree
  - Players alternate
  - Choose move to position with highest minimax value = best achievable utility against best play



Game Search

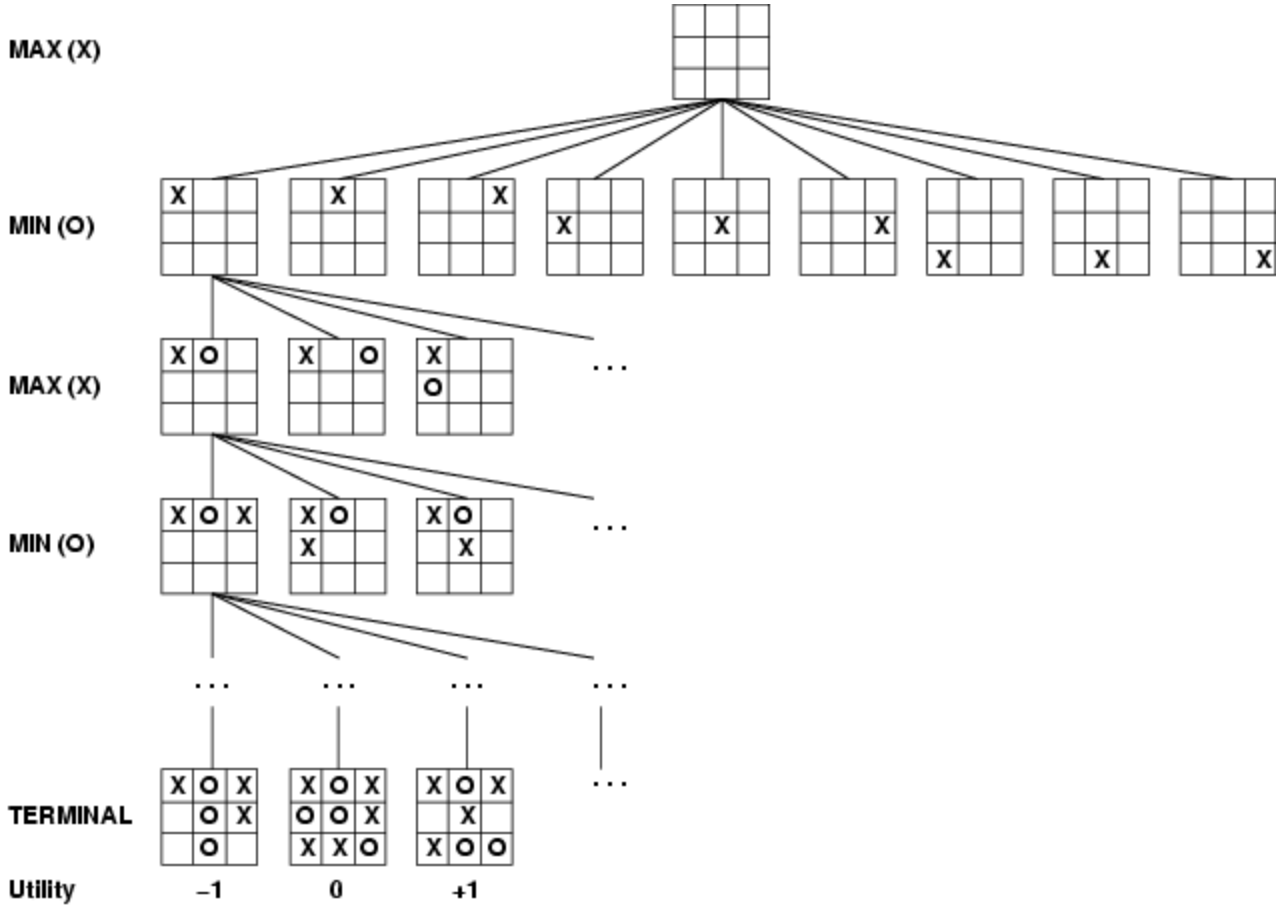
- Problem Formulation
  - **Initial state**: initial board position + information about whose move it is
  - **Successors**: legal moves a player can make
  - **Goal (terminal test):** determines when the game is over
  - **Utility function**: measures the outcome of the game and its desirability

- Search objective
  - Find the sequence of player's decisions (moves) maximizing its utility
  - Consider the opponent's moves and their utility

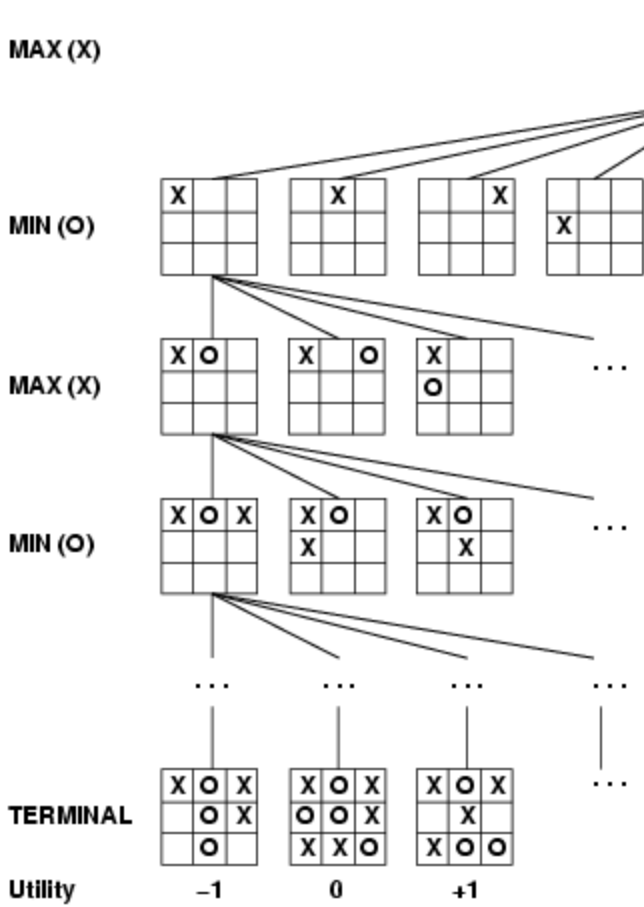
Game Tree

- Initial State and Legal Moves for Each Side

Game Tree
(2-player, deterministic, turns)

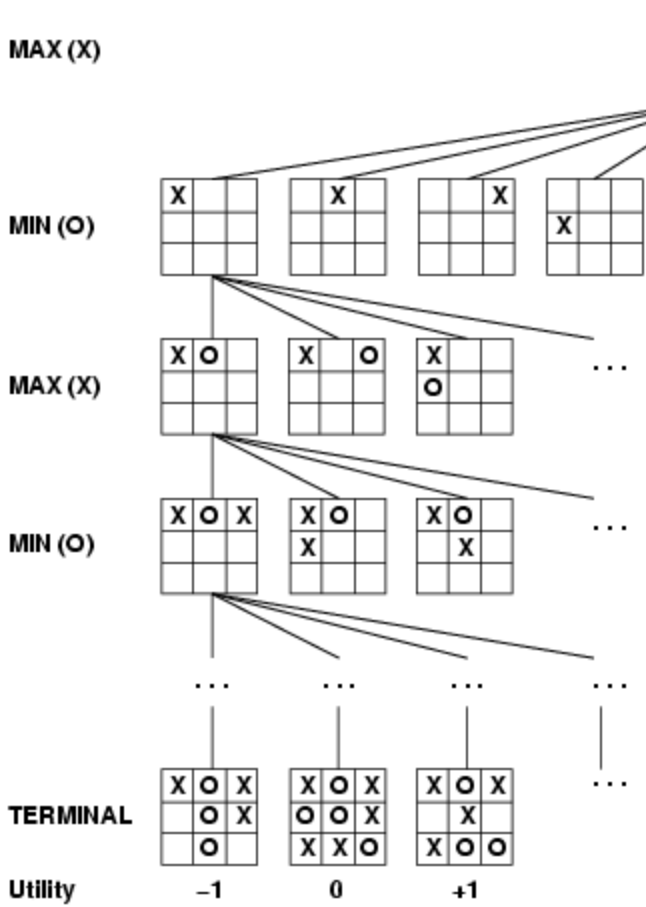


Game Tree (2-player, deterministic, turns)



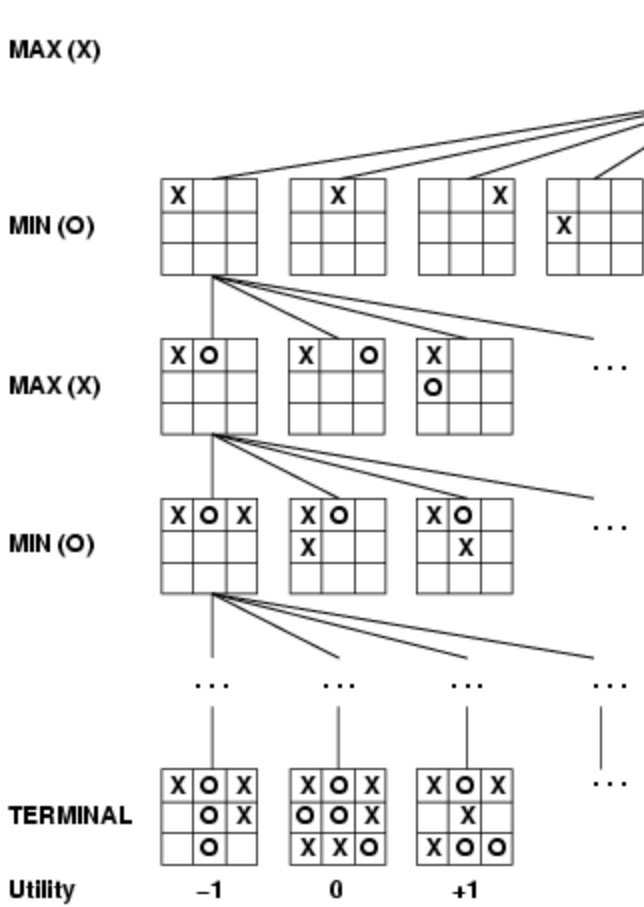
- *MAX and MIN are the 2 players*
- *MAX goes first*
- *Players then take turns*

Game Tree (2-player, deterministic, turns)



- *MAX has 9 possible legal first moves (ignoring symmetry)*

Game Tree (2-player, deterministic, turns)



- *Utility of terminal states (when game is over) is from MAX's point of view*
- *Points are awarded to both players at the end of the game*
  - *-1 is a loss*
  - *0 is a draw*
  - *1 is a win*

Minimax Algorithm

- How do we deal with the contingency problem?
  - Assuming that the opponent is rational and always optimizes its behavior (opposite to us), we consider the opponent's best response
  - Then the minimax algorithm determines the best move

Minimax

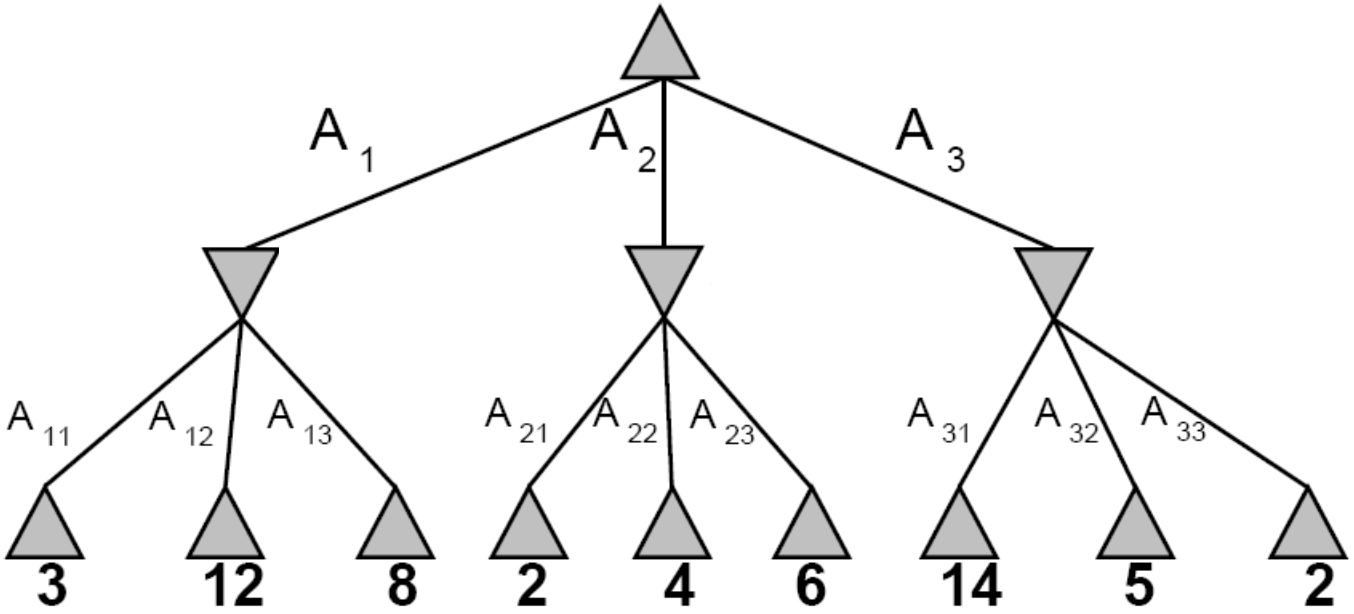
- Finds an optimal (contingent) strategy, assuming perfect play for deterministic games

- Idea: choose move to position with highest MINIMAX VALUE
  = best achievable payoff against best play

- MINIMAX-VALUE (*n*)
  - UTILITY (*n*) if *n* is a terminal state
  - max_*s* MINIMAX-VALUE (*s*) if *n* is a MAX node
  - min_*s* MINIMAX-VALUE (s) if *n* is a MIN node

  (where *s is an element of the successors of n)*

Minimax Example



Properties of minimax

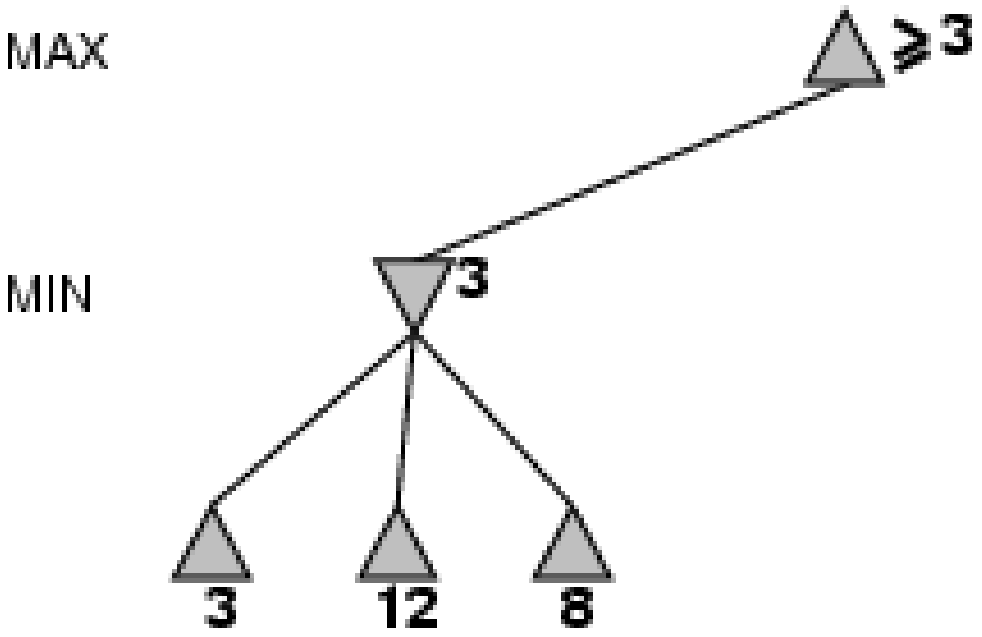
- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible
- Do we really need to explore every path???

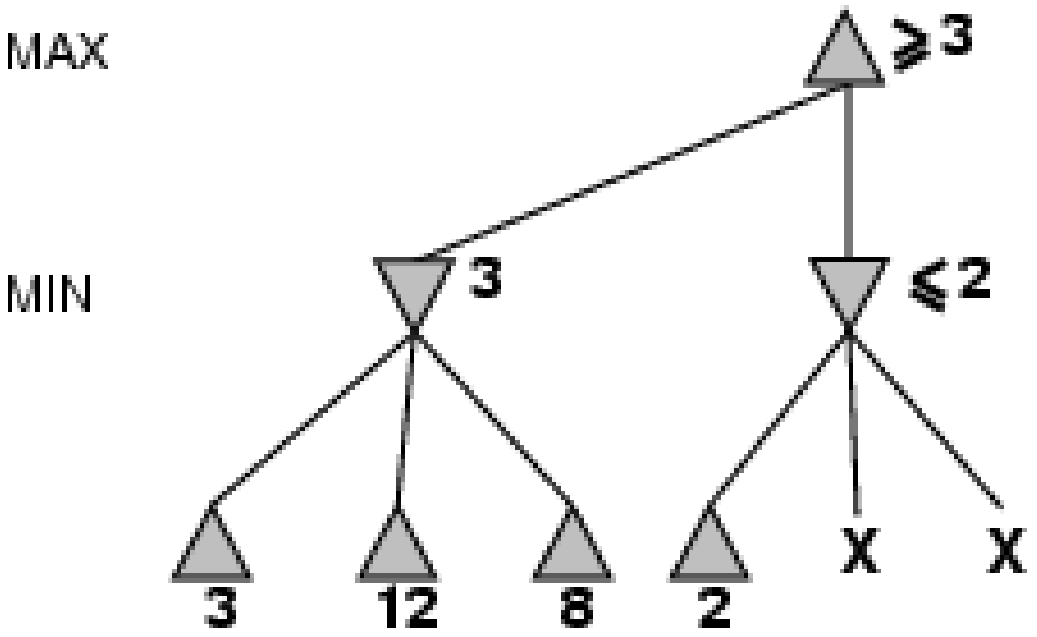
Solutions to the Complexity Problem

- Dynamic pruning of redundant branches of the search tree
  - Some branches will never be played by rational players since they include sub-optimal decisions (for either player)
    - Identify a provably suboptimal branch of the search tree before it is fully explored
    - Eliminate the suboptimal branch
  - Procedure: Alpha-Beta Pruning
- Early cutoff of the search tree
  - Use imperfect minimax value estimate of non-terminal states

α-β pruning example



α-β pruning example



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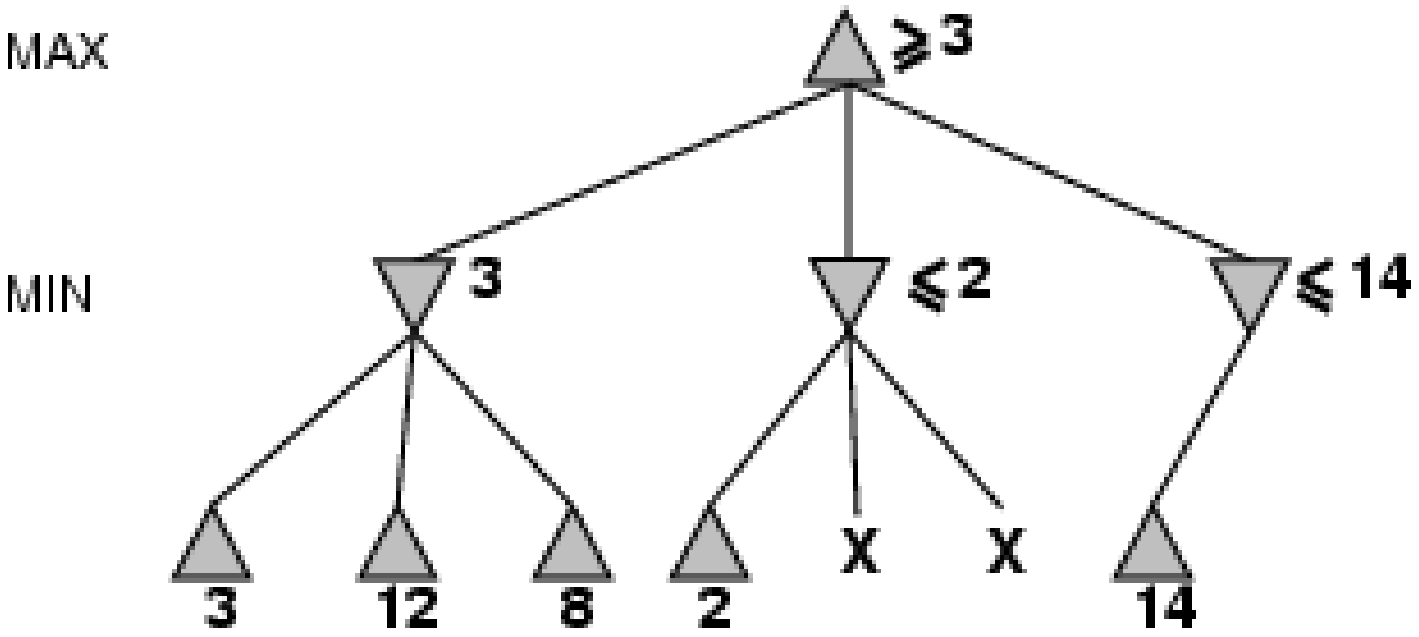

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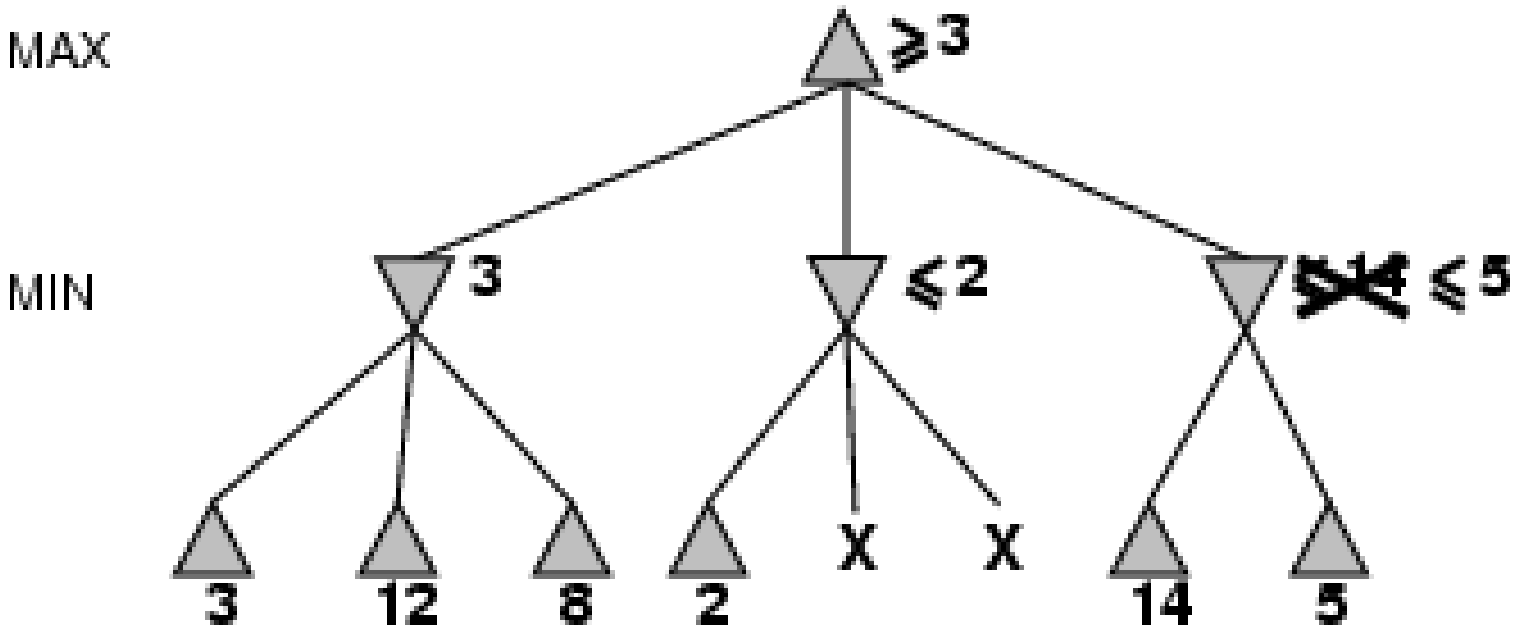

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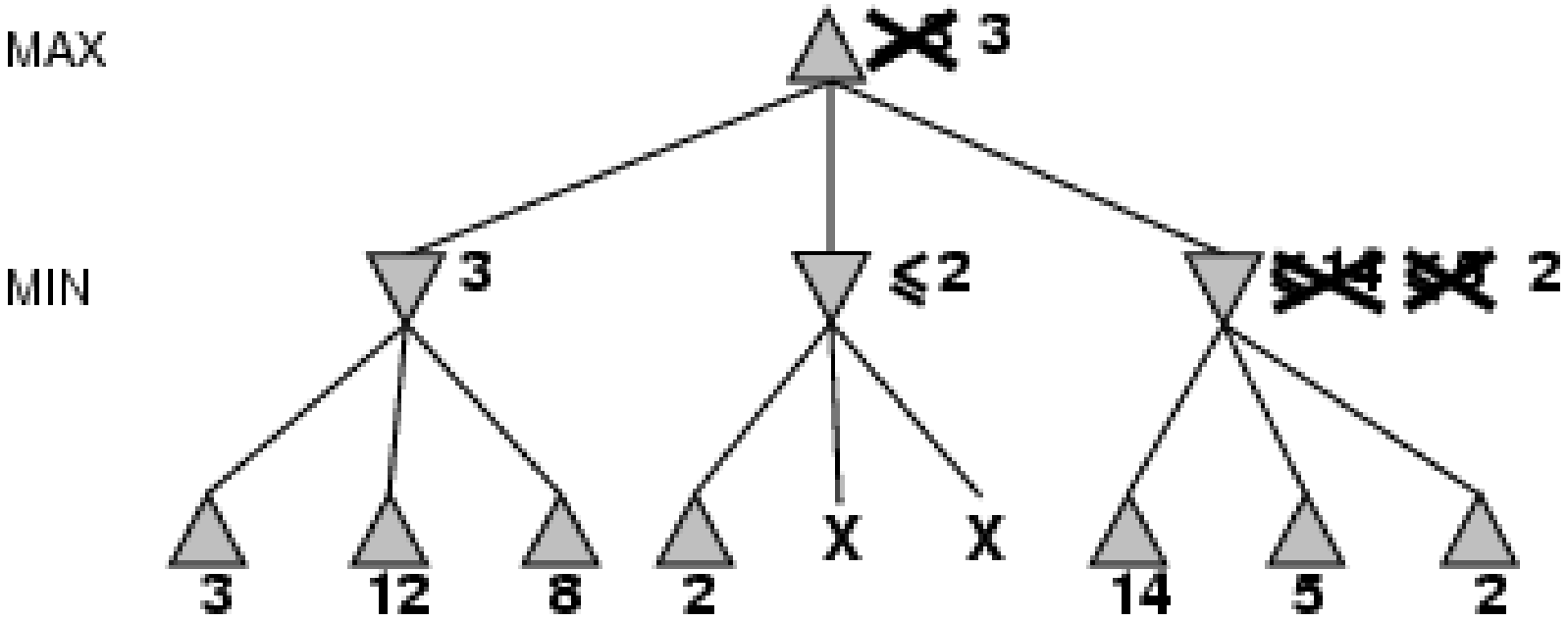
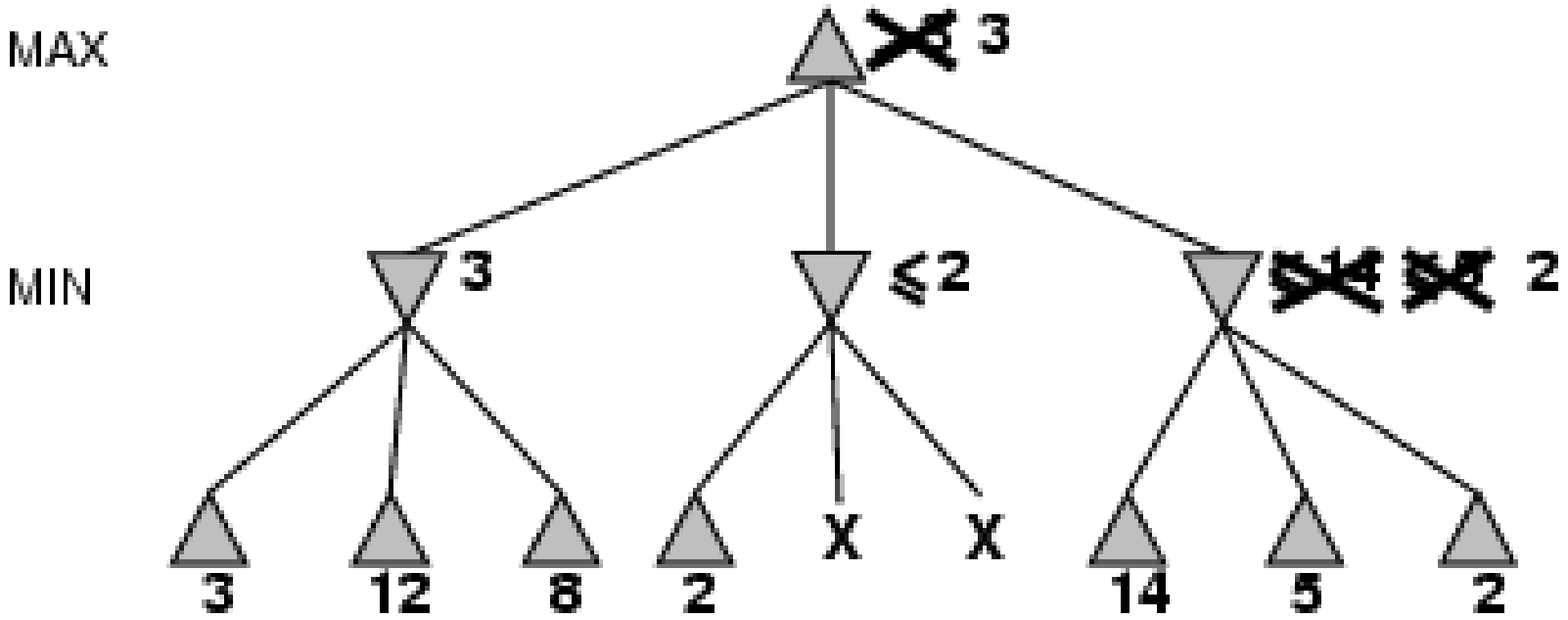


α-β pruning example



MINIMAX-VALUE(root)
= max(min(3,12,8), min(2,x,y), min(14,5,2))
= max (3, min(2,x,y), 2)
= max(3, z, 2) for z <= 2
= 3

Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Resource limits

Recap

- Minimax explores the full search space
- Alpha Beta prunes, but still searches all the way to terminal states for a portion of the search space

Standard approaches to fix resource limits

- cutoff test:

  e.g., depth limit

- evaluation function

  = estimated desirability of position

Evaluation functions

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with

  $f_1(s)$ = (number of white queens) – (number of black queens), etc.

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

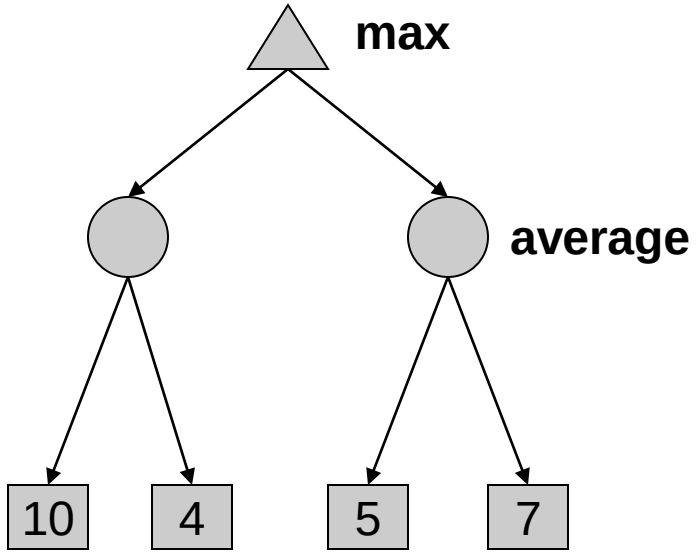
4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov

Stochastic Single-Player

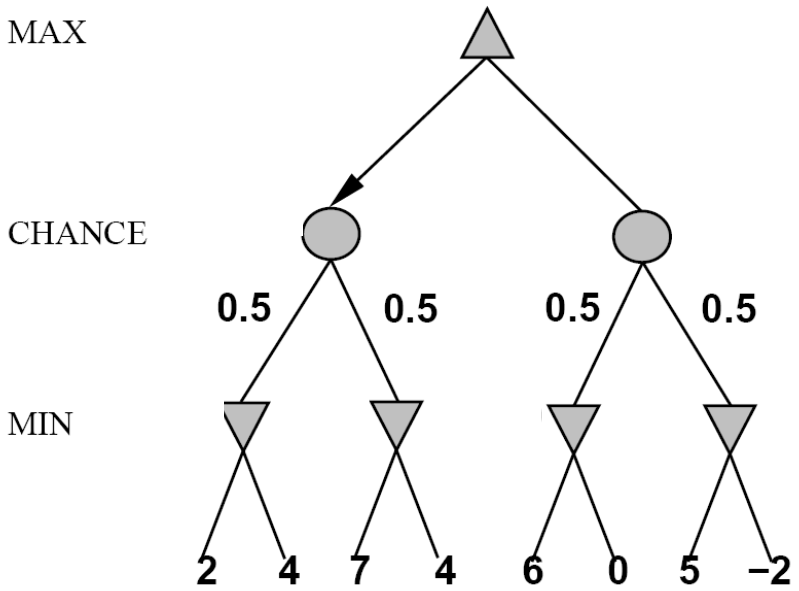
- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, mine locations
  - In pacman, ghosts!

- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Calculate utility for each node
  - Max nodes as in search
  - Chance nodes take average (expectation) of value of children



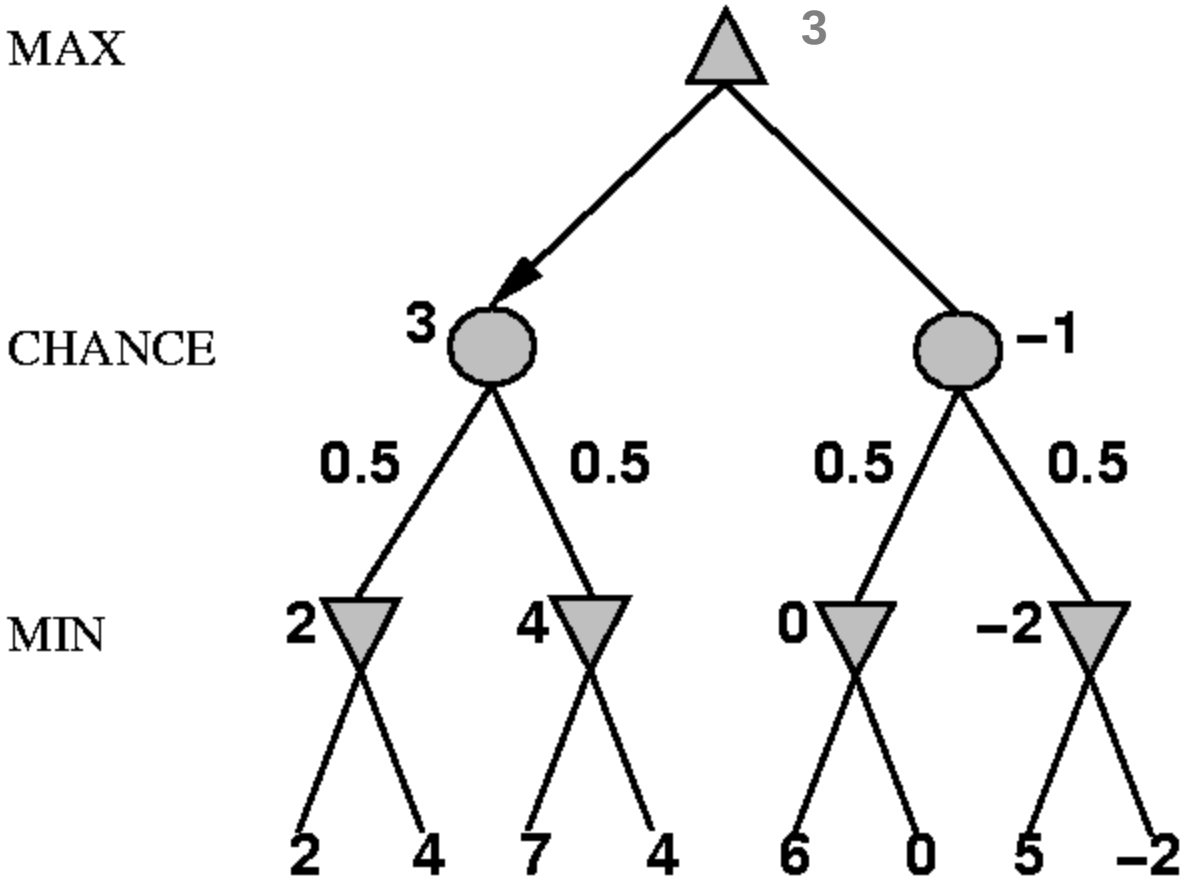
Stochastic Two-Player

- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



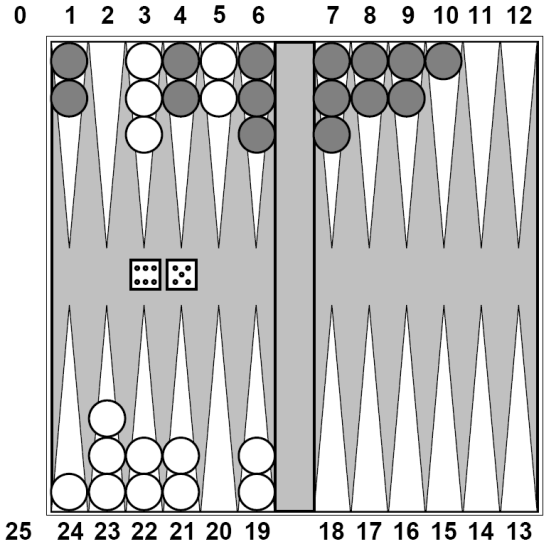
if *state* is a MAX node **then**
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Minimax for nondeterministic games



Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about