CS2710/ISSP2160
Homework 7 Solutions

13.8 The main point of this exercise is to understand the various notations of bold versus non-bold P, and uppercase versus lowercase variable names. The rest is easy, involving a small matter of addition.

a. This asks for the probability that *Toothache* is true.

$$P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$$

b. This asks for the vector of probability values for the random variable *Cavity*. It has two values, which we list in the order $\langle true, false \rangle$. First add up $0.108 + 0.012 + 0.072 + 0.008 = 0.2$. Then we have

$$\mathbf{P}(Cavity) = \langle 0.2, 0.8 \rangle \ .$$

c. This asks for the vector of probability values for *Toothache*, given that *Cavity* is true.

$$\mathbf{P}(Toothache|cavity) = \langle (.108 + .012)/0.2, (0.072 + 0.008)/0.2 \rangle = \langle 0.6, 0.4 \rangle$$

d. This asks for the vector of probability values for *Cavity*, given that either *Toothache* or *Catch* is true. First compute $P(toothache \lor catch) = 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 = 0.416$. Then

$$\mathbf{P}(Cavity|toothache \lor catch) =$$
$$\langle (0.108 + 0.012 + 0.072)/0.416, (0.016 + 0.064 + 0.144)/0.416 \rangle =$$
$$\langle 0.4615, 0.5384 \rangle$$

2: Probability

Let D denote the event "having the disease" and let + denote the event "test positive"

We are given the following information:

P(D) = 0.02
which implies P(not D) = 0.98
P(not + | D) = 0.06
which implies P(+ | D) = 0.94
P(+ | not D) = 0.09
First, we compute P(+)
= P(+ AND D) + P(+ AND (not D))
= P(+ | D) P(D) + P(+ | not D) P(not D)
= 0.94 x 0.02 + 0.09 x 0.98
= 0.107

We would like to know P(D | +)

= P(+ | D) x P(D) / P(+)
= 0.94 x 0.02 / 0.107
~= 0.1757

14.8 Adding variables to an existing net can be done in two ways. Formally speaking, one should insert the variables into the variable ordering and rerun the network construction process from the point where the first new variable appears. Informally speaking, one never really builds a network by a strict ordering. Instead, one asks what variables are direct causes or influences on what other ones, and builds local parent/child graphs that way. It is usually easy to identify where in such a structure the new variable goes, but one must be very careful to check for possible induced dependencies downstream.

a. $IcyWeather$ is not caused by any of the car-related variables, so needs no parents. It directly affects the battery and the starter motor. $StarterMotor$ is an additional precondition for $Starts$. The new network is shown in Figure S14.1.

b. Reasonable probabilities may vary a lot depending on the kind of car and perhaps the personal experience of the assessor. The following values indicate the general order of

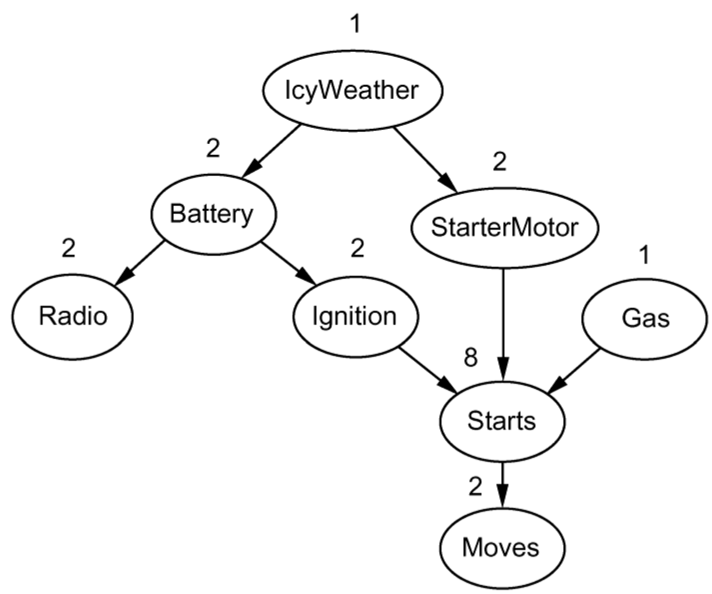

Figure S14.1 Car network amended to include $IcyWeather$ and $StarterMotorWorking$ (SMW).

magnitude and relative values that make sense:

- A reasonable prior for IcyWeather might be 0.05 (perhaps depending on location and season).
- $P(Battery|IcyWeather) = 0.95$, $P(Battery|\neg IcyWeather) = 0.997$.
- $P(StarterMotor|IcyWeather) = 0.98$, $P(Battery|\neg IcyWeather) = 0.999$.
- $P(Radio|Battery) = 0.9999$, $P(Radio|\neg Battery) = 0.05$.
- $P(Ignition|Battery) = 0.998$, $P(Ignition|\neg Battery) = 0.01$.
- $P(Gas) = 0.995$.
- $P(Starts|Ignition, StarterMotor, Gas) = 0.9999$, other entries 0.0.
- $P(Moves|Starts) = 0.998$.

c. With 8 Boolean variables, the joint has $2^8 - 1 = 255$ independent entries.

d. Given the topology shown in Figure S14.1, the total number of independent CPT entries is 1+2+2+2+2+1+8+2= 20.

e. The CPT for $Starts$ describes a set of nearly necessary conditions that are together almost sufficient. That is, all the entries are nearly zero except for the entry where all the conditions are true. That entry will be not quite 1 (because there is always some other possible fault that we didn't think of), but as we add more conditions it gets closer to 1. If we add a $Leak$ node as an extra parent, then the probability is exactly 1 when all parents are true. We can relate noisy-AND to noisy-OR using de Morgan's rule: $A \land B \equiv \neg(\neg A \lor \neg B)$. That is, noisy-AND is the same as noisy-OR except that the polarities of the parent and child variables are reversed. In the noisy-OR case, we have

$$P(Y = true|x_1, \ldots, x_k) = 1 - \prod_{\{i:x_i = true\}} q_i$$

where q_i is the probability that the *presence* of the ith parent *fails* to cause the child to be *true*. In the noisy-AND case, we can write

$$P(Y = true|x_1, \ldots, x_k) = \prod_{\{i:x_i = false\}} r_i$$

where r_i is the probability that the *absence* of the ith parent *fails* to cause the child to be *false* (e.g., it is magically bypassed by some other mechanism).

In addition:

P(Battery = T) × P(Radio = T|Battery = T) × P(Ignition = T|Battery = T) × P(Gas = F) × P(Starts = T|Gas = F, Ignition = T) ×P(Moves = F|Starts = T)

4. More Bayesian Networks

(blind approach: 5) $P(M = F) = \sum_{b\,\{T,F\}} \sum_{r\,\{T,F\}} \sum_{i\{T,F\}} \sum_{g\{T,F\}} \sum_{s\{T,F\}}$
$P(B = b) \times P(R = r|B = b) \times P(I = i|B = b) \times P(G = g) \times P(S = s|G = g, I = i) \times P(M = F|S = s)$

One way to interleave the expressions is as follows:

$P(M = F) = \sum_{b\,\{T,F\}} P(B = b)$

$X\ [\sum_{r\,\{T,F\}} P(R = r|B = b)\ ]$

$X\ [\sum_{i\,\{T,F\}} P(I = i|B = b)\ \ x\ \{ \sum_{g\{T,F\}} P(G = g) \times (\sum_{s\,\{T,F\}} P(S = s|G = g, I = i) \times P(M = F|S = s)\}]$

We can directly eliminate the sums over Radio values since they must add up to 1

5. Diagnosis using Bayesian Networks

The diagnostic probability can be rewritten as the ratio of two joint probabilities:

$P(Pn = T|Fe = T, Pa = F, Cou = T,HWBC = F)$
$=$
$P(Pn = T, Fe = T, Pa = F, Cou = T,HWBC = F) /\ \ P(Fe = T, Pa = F, Cou = T,HWBC = F)$

The numerator of the fraction is the full joint probability and thus can be rewritten as a product of conditionals:

$P(Pn = T, Fe = T, Pa = F, Cou = T,HWBC = F)$
$=$
$P(Pn = T)P(Fe = T|Pn = T)P(Pa = F|Pn = T)P(Cou = T|Pn = T)P(HWBC = F|Pn = T)$

On the other hand the denominator is not a full joint so we express the probability in terms of the sum of the full joint probabilities, and second we express the full joint probabilities as the product of the belief network conditionals:

$P(Fe = T, Pa = F, Cou = T,HWBC = F)$
$=$
$\sum_{p\,\{T,F\}} P(Pn = p, Fe = T, Pa = F, Cou = T,HWBC = F)$
$=$
$\sum_{p\{T,F\}} P(Pn = p)P(Fe = T|Pn = p)P(Pa = F|Pn = p)P(Cou = T|Pn = p)P(HWBC = F|Pn = p)$

This makes it possible to compute the queries directly from the probability distribution tables provided above. That is:

$P(Pn = T|Fe = T, Pa = F, Cou = T,HWBC = F)$
$=$
$(0.02 \times 0.9 \times 0.3 \times 0.9 \times 0.2) / (0.02 \times 0.9 \times 0.3 \times 0.9 \times 0.2 + 0.98 \times 0.6 \times 0.5 \times 0.1 \times 0.5)$
$= 0.062021$

6. JavaBayes

The diagnostic probability can be rewritten as the ratio of two joint probabilities:

1.
    a. .9329
    b. P(Lights | BatteryPower=Good) = .9
       P(Lights | BatteryPower=Poor) = 0.0
    c. .5889

2
    a. .9995
    b. .9532
    c. .9995
    d. .9997

3

P(Radio = Dead, Lights = NoLight|BatteryPower = Good)
= P(Radio = Dead|Lights = NoLight, BatteryPower = Good) x P(Lights = NoLight|BatteryPower = Good)
= 0.1 x 0.1 = 0.01