1a

7.1 To save space, we'll show the list of models as a table rather than a collection of diagrams. There are eight possible combinations of pits in the three squares, and four possibilities for the wumpus location (including nowhere).

We can see that $KB \models \alpha_2$ because every line where KB is true also has α_2 true. Similarly for α_3.

Model	KB	α_2	α_3
		true	
$P_{1,3}$		*true*	
$P_{2,2}$			
$P_{3,1}$		*true*	
$P_{1,3}, P_{2,2}$			
$P_{2,2}, P_{3,1}$			
$P_{3,1}, P_{1,3}$		*true*	
$P_{1,3}, P_{3,1}, P_{2,2}$			
$W_{1,3}$		*true*	*true*
$W_{1,3}, P_{1,3}$		*true*	*true*
$W_{1,3}, P_{2,2}$			*true*
$W_{1,3}, P_{3,1}$	*true*	*true*	*true*
$W_{1,3}, P_{1,3}, P_{2,2}$			*true*
$W_{1,3}, P_{2,2}, P_{3,1}$			*true*
$W_{1,3}, P_{3,1}, P_{1,3}$		*true*	*true*
$W_{1,3}, P_{1,3}, P_{3,1}, P_{2,2}$			*true*
$W_{3,1}$,		*true*	
$W_{3,1}, P_{1,3}$		*true*	
$W_{3,1}, P_{2,2}$			
$W_{3,1}, P_{3,1}$		*true*	
$W_{3,1}, P_{1,3}, P_{2,2}$			
$W_{3,1}, P_{2,2}, P_{3,1}$			
$W_{3,1}, P_{3,1}, P_{1,3}$		*true*	
$W_{3,1}, P_{1,3}, P_{3,1}, P_{2,2}$			
$W_{2,2}$		*true*	
$W_{2,2}, P_{1,3}$		*true*	
$W_{2,2}, P_{2,2}$			
$W_{2,2}, P_{3,1}$		*true*	
$W_{2,2}, P_{1,3}, P_{2,2}$			
$W_{2,2}, P_{2,2}, P_{3,1}$			
$W_{2,2}, P_{3,1}, P_{1,3}$		*true*	
$W_{2,2}, P_{1,3}, P_{3,1}, P_{2,2}$			

Figure 7.1 A truth table constructed for Ex. 7.1. Propositions not listed as true on a given line are assumed false, and only *true* entries are shown in the table.

1b

```
> (truth-table "P ^ Q <=> Q ^ P")
-----------------------------------------------
 P  Q  P ^ Q  Q ^ P  (P ^ Q) <=> (Q ^ P)
-----------------------------------------------
 F  F    F      F          \(true\)
 T  F    F      F             T
 F  T    F      F             T
 T  T    T      T             T
-----------------------------------------------
```

$a \Rightarrow b \equiv$
$\sim a \vee b \equiv$ implication elim
$b \vee \sim a \equiv$ commutativity
$\sim\sim b \vee \sim a \equiv$ double neg
$\sim b \Rightarrow \sim a$ implication elim

2.

(a) Let P: I have a sweet tooth, Q: I like chocolate, R: I like cake, S: I like danish, T: I'm chocoholic.
KB: $(P \wedge Q) \vee (Q \wedge R)$
Rules: $R \Rightarrow S$, $S \Rightarrow P$, $(P \wedge Q) \Rightarrow T$
Goal: T.

(b) CNF representation
KB:

- $(P \vee Q) \wedge (P \vee R) \wedge Q \wedge (Q \vee R)$ (By application of the distributive property). Using the absorption law, this is equivalent to:
- $(P \vee R) \wedge Q$

Rules:

- $(\neg R \vee S) \wedge (\neg S \vee P) \wedge (\neg(P \wedge Q) \vee T)$
- $(\neg R \vee S) \wedge (\neg S \vee P) \wedge (\neg P \vee \neg Q \vee T)$ (De Morgan's law, associativity)

Goal: T

(c) Resolution: We rewrite the knowledge base as a sequence of disjunctive sentences (the conjunction connective is implicit) ending with the negated theorem:

1. $(P \vee R)$
2. Q
3. $(\neg R \vee S)$
4. $(\neg S \vee P)$
5. $(\neg P \vee \neg Q \vee T)$
6. $\neg T$
Now we apply resolution: Resolving 5 and 6 yields

5'. $(\neg P \vee \neg Q)$
Resolving 3 and 4 yields

3'. $\neg R \vee P$
1 and 3' give:

1'. P
1' and 5' give:

1". : $\neg Q$
Resolving 1" and 2 results in a contradiction thus proving the theorem T.

3a

f. No person buys an expensive policy.
$\forall x, y, z \;\; Person(x) \wedge Policy(y) \wedge Expensive(y) \;\Rightarrow\; \neg Buys(x, y, z)$.

g. ~~There is an agent~~ who sells policies only to people who are not insured.
$\exists x \;\; Agent(x) \wedge \forall y, z \;\; Policy(y) \wedge Sells(x, y, z) \;\Rightarrow\; (Person(z) \wedge \neg Insured(z))$.

h. There is a barber who shaves all men in town who do not shave themselves.
$\exists x \;\; Barber(x) \wedge \forall y \;\; Man(y) \wedge \neg Shaves(y, y) \;\Rightarrow\; Shaves(x, y)$.

3b

$GrandChild(c, a) \;\Leftrightarrow\; \exists b \;\; Child(c, b) \wedge Child(b, a)$

3c **9.4** This is an easy exercise to check that the student understands unification.

a. $\{x/A, y/B, z/B\}$ (or some permutation of this).

b. No unifier (x cannot bind to both A and B).

c. $\{y/John, x/John\}$.

d. No unifier (because the occurs-check prevents unification of y with $Father(y)$).

---

4 (a) Knowledge base:

1. $Programmer(x) \land Emulator(y) \land People(z) \land Provide(x, y, z) \implies Criminal(x)$
2. $Use(Friends, x) \land Runs(x, GameXgames) \implies Provide(SuperProgrammer, x, Friends)$
3. $Software(x) \land Runs(x, GameXgames) \implies Emulator(x)$
4. $Programmer(SuperProgrammer)$
5. $People(Friends)$
6. $Software(Emulator1)$
7. $Use(Friends, Emulator1)$
8. $Runs(Emulator1, GameXgames)$

(b) Forward chaining:

Step 1: Rules 2 and 3 can fire, but rule 2 fires first since it has a lower rule number. (Facts 8 and 7 match the premise of rule 2 with Emulator1 substituted for x). Firing rule 2 adds the fact $Provide(SuperProgrammer, Emulator1, Friends)$ to the knowledge base. Rule 2 will not fire again with these bindings.

Step 2: Rule 3 fires since facts 6 and 8 match its premise with Emulator1 substituted for x, adding the fact $Emulator(Emulator1)$ to the knowledge base.

Step 3: Rule 1 fires since facts 4 and 5 and the newly generated facts $Provide(SuperProgrammer, Emulator1, Friends)$ and $Emulator(Emulator1)$ match this premise with SuperProgrammer substituted for x, Emulator1 for y, and Friends for z, adding the fact $Criminal(SuperProgrammer)$ to the knowledge base.

(c) Backward chaining:
We want to prove $Criminal(SuperProgrammer)$. It is not in the knowledge base. It matches

the conclusion of only rule 1 with SuperProgrammer substituted for x. Thus, we need to prove the premise of rule 1, namely $Programmer(SuperProgrammer)$, $Emulator(y)$, $People(z)$, and $Provide(SuperProgrammer, y, z)$, for some objects y and z. We consider these in turn.

$Programmer(SuperProgrammer)$ is proven since it is in the knowledge base.

$Emulator(y)$ does not match facts in the knowledge base but it matches the conclusion of rule 3 with y equal to x. Thus, we need to prove the premise of rule 3, namely $Software(y)$ and $Runs(y, GameXgames)$. $Software(y)$ is proven with Emulator1 substituted for y since $Software(Emulator1)$ is in the knowledge base. We then need to prove $Runs(Emulator1, GameXgames)$. This is proven since it is in the knowledge base as well.

$People(z)$ is proven with Friends substituted for z since $People(Friends)$ is in the knowledge base.

Finally, $Provide(SuperProgrammer, y, z)$ now is $Provide(SuperProgrammer, Emulator1, Friends)$, which is not in the knowledge base but matches the conclusion of rule 2 with Emulator1 substituted for x. Thus, we need to prove the premise of rule 2, namely $Use(Friends, Emulator1)$ and $Runs(Emulator1, GameXgames)$, which are both in the knowledge base.

At this point, the proof of $Criminal(SuperProgrammer)$ succeeds.

5.

1. Anyone passing his history exams and winning the lottery is happy

$\forall x: pass(x, history) \wedge win(x, lottery) \rightarrow happy(x)$

2. But anyone who studies or is lucky can pass all his exams

$\forall x \forall y: study(x) \vee lucky(x) \rightarrow pass(x, y)$

3. John did not study but he is lucky

$\neg study(John) \wedge lucky(John)$

4. Anyone who is lucky wins the lottery

$\forall x: lucky(x) \rightarrow win(x, lottery)$

5. There exists a person who is wealthy

$\exists x: wealthy(x)$

Goal:

happy(John)

(b) Conjunctive Normal Form ([redacted])

1.

$\forall x: pass(x, history) \wedge win(x, lottery) \rightarrow happy(x)$

$= \forall x: \neg[pass(x, history) \wedge win(x, lottery)] \vee happy(x)$

$= \forall x: \neg pass(x, history) \vee \neg win(x, lottery) \vee happy(x)$

2.

$\forall x \forall y: study(x) \vee lucky(x) \rightarrow pass(x, y)$

$= \forall x \forall y: \neg[study(x) \vee lucky(x)] \vee pass(x, y)$

$= \forall x \forall y: [\neg study(x) \wedge \neg lucky(x)] \vee pass(x, y)$

$= \forall x \forall y: (\neg study(x) \vee pass(x, y)) \wedge (\neg lucky(x) \vee pass(x, y))$

Two CNF:

$\forall x \forall y: \neg study(x) \vee pass(x, y)$

$\forall x \forall y: \neg lucky(x) \vee pass(x, y)$

3. Two CNF

$\neg study(John)$

$lucky(John)$

4.

$\forall x: \neg lucky(x) \vee win(x, lottery)$

5. *wealthy(skolem)*

(c) Resolution by refutation ([redacted])

1 $\forall x: \neg pass(x, history) \vee \neg win(x, lottery) \vee happy(x)$

2 $\forall x \forall y: \neg lucky(x) \vee pass(x, y)$

3 $lucky(John)$

4 $\forall x: \neg lucky(x) \vee win(x, lottery)$

5 $\neg happy(John)$ negation of the goal

Note: only part of facts are used

6. $\neg pass(John, history) \vee \neg win(John, lottery)$	1 and 5, x is replaced by *John*
7. $\forall y: pass(John, y)$	2 and 3, x is replaced by *John*
8. $win(John, lottery)$	3 and 4, x is replaced by *John*
9. $\neg win(John, lottery)$	6 and 7, y is replaced by *history*
10 *empty*	8 and 9