CS/COE 1501

www.cs.pitt.edu/~lipschultz/cs1501/

Union Find

Dynamic connectivity problem

- For a given graph G, can we determine whether or not two vertices are connected in G?
- Can also be viewed as checking subset membership
- Important for many practical applications
- We will solve this problem using a union/find data structure

Union Find API

Initialize with n items numbered 0 to n-1

```
UF (int n)
```

Connect p with q

```
void union(int p, int q)
```

Return id of the connected component that p is in

```
int find (int p)
```

```
boolean connected (int p, int q)
```

True if p and q are connected

```
int count()
```

Number of connected components

Covering the basics

```
public int count() {
    return count;
}

public boolean connected(int p, int q) {
    return find(p) == find(q);
}
```

A simple approach

- Have an *id* array simply store the component id for each item in the union/find structure
  - Find simply returns its id
  - What about union?

Example

U(2, 0)

U(4, 7)

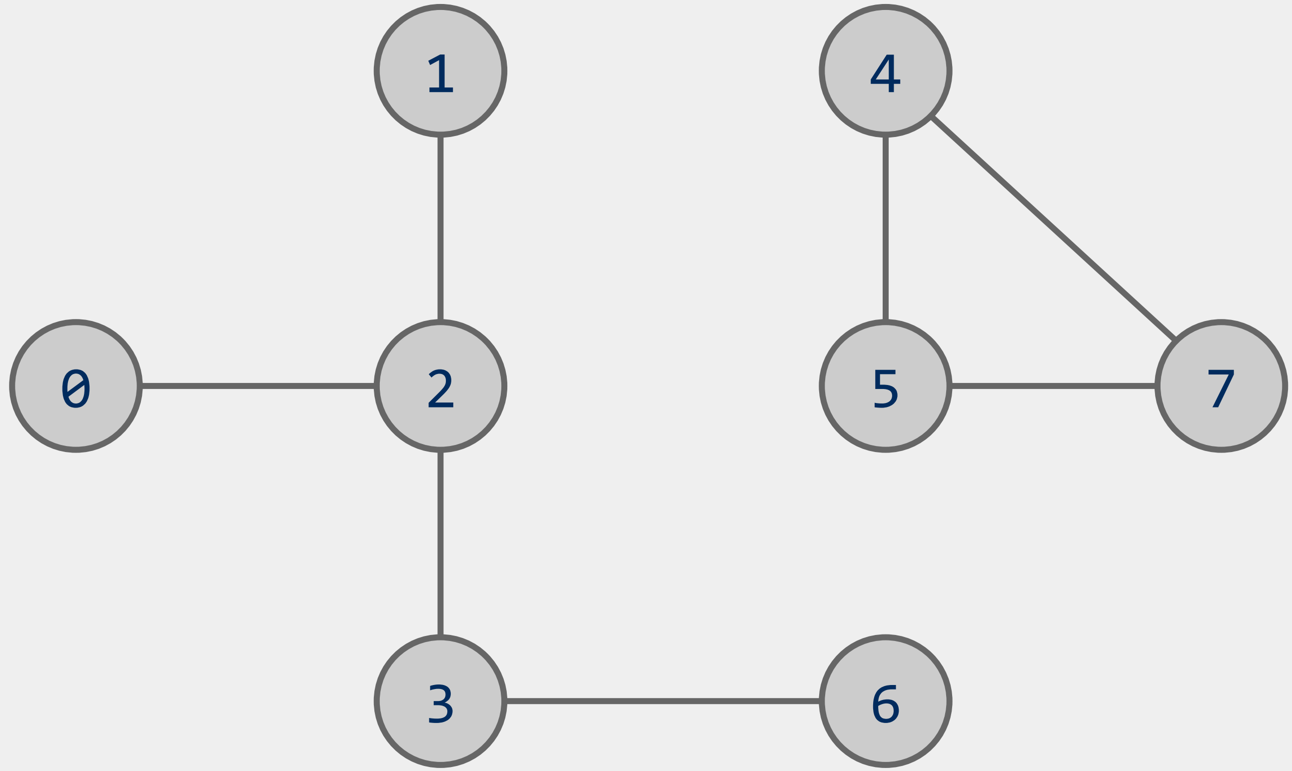
U(1, 2)

U(3, 2)

U(4, 5)

U(5, 7)

U(6, 3)



	0	1	2	3	4	5	6	7
ID:	6	6	6	6	4	4	6	4

Implementing the basic approach

```
public UF(int n) {
    count = n;
    id = new int[n];
    for (int i = 0; i < n; i++) { id[i] = i; }
}

public int find(int p) { return id[p]; }

public void union(int p, int q) {
    int pID = find(p), qID = find(q);
    if (pID == qID) return;
    for(int i = 0; i < id.length; i++)
        if (id[i] == pID) id[i] = qID;
    count--;
}
```

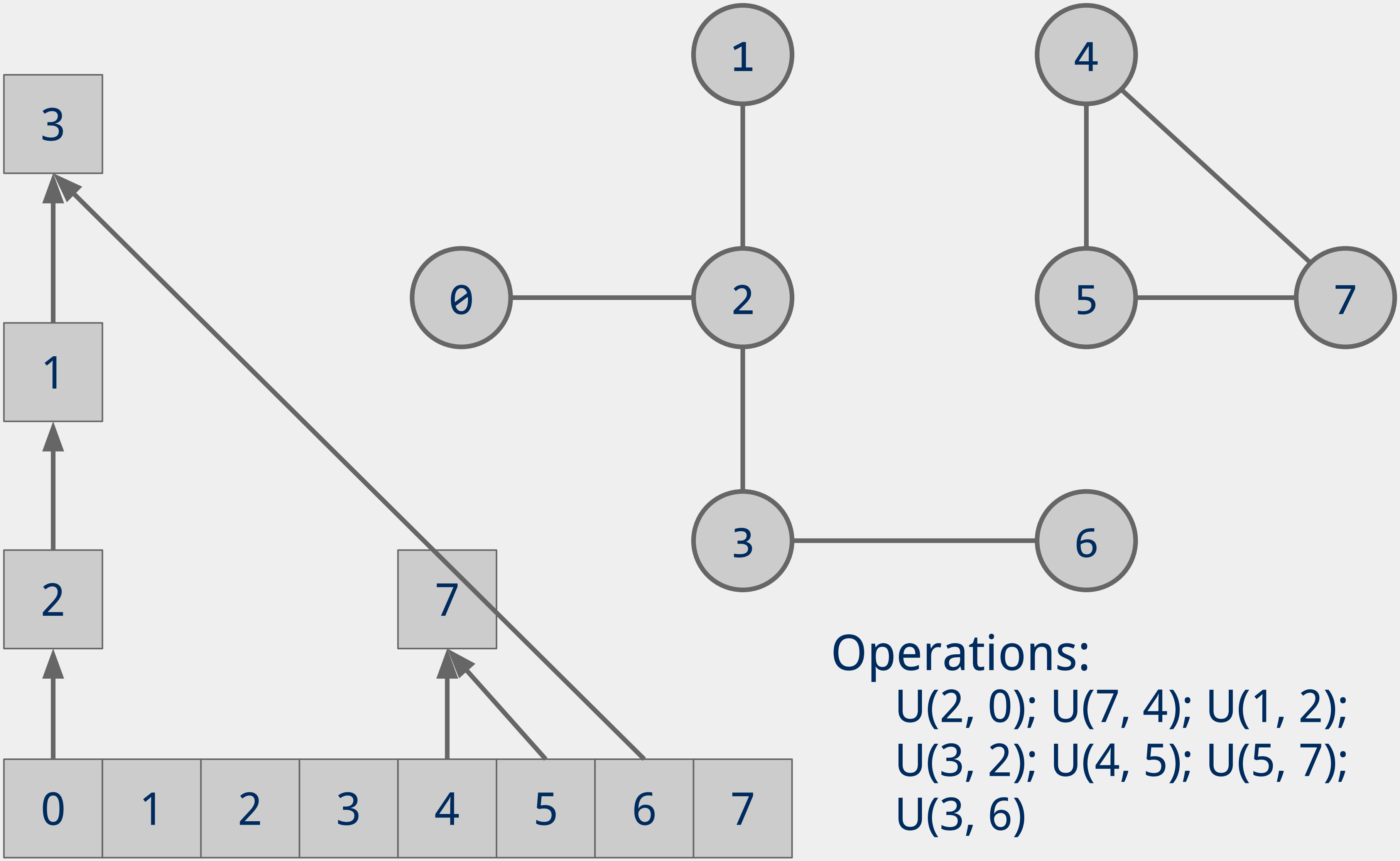
Analysis of our simple approach

- Runtime?
  - For find():
    - $\Theta(1)$
  - For union():
    - $\Theta(n)$

Can we improve on union()'s runtime?

- What if we store our connected components as a forest of trees?
  - Each tree representing a different connected component
  - Every time a new connection is made, we simply make one tree the child of another

Tree example



Operations:

U(2, 0); U(7, 4); U(1, 2);
U(3, 2); U(4, 5); U(5, 7);
U(3, 6)

Implementation using the same id array

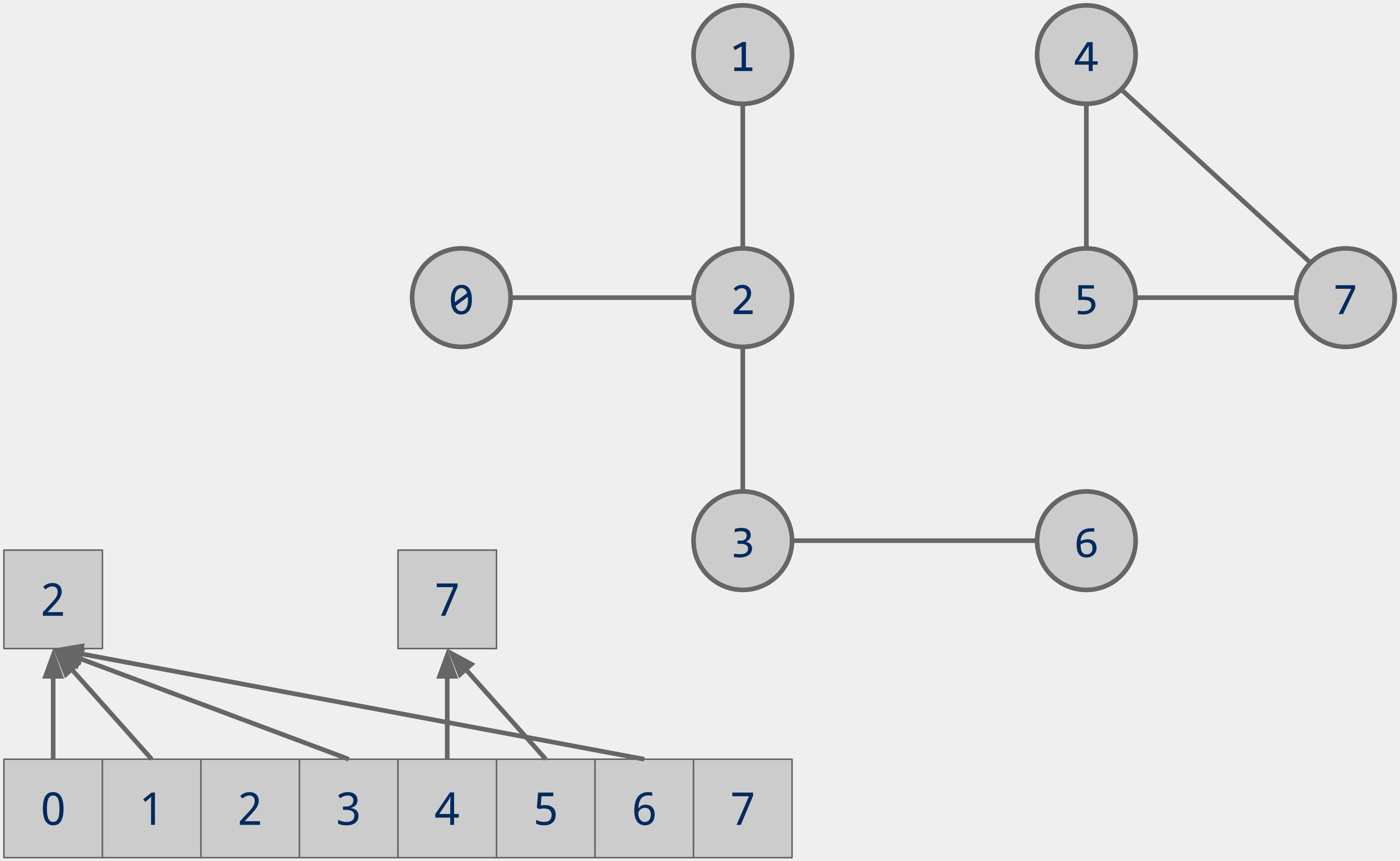
```
public int find(int p) {
    while (p != id[p]) p = id[p];
    return p;
}

public void union(int p, int q) {
    int i = find(p);
    int j = find(q);
    if (i == j) return;
    id[j] = i;
    count--;
}
```

Forest of trees implementation analysis

- Runtime?
    - find():
        - Bound by the height of the tree
    - union():
        - Bound by the height of the tree
- What is the max height of the tree?
    - Can we modify our approach to cap its max height?

Weighted tree example



Weighted trees

```
public UF(int n) {
    count = n;
    id = new int[n];
    sz = new int[n];
    for (int i = 0; i < n; i++) { id[i] = i; sz[i] = 1; }
}

public void union(int p, int q) {
    int i = find(p), j = find(q);
    if (i == j) return;
    if (sz[i] < sz[j]) { id[i] = j; sz[j] += sz[i]; }
    else               { id[j] = i; sz[i] += sz[j]; }
    count--;
}
```

Weighted tree approach analysis

- Runtime?
  - find():
    - $\Theta(\log n)$
  - union():
    - $\Theta(\log n)$

- Can we do any better?

Kruskal’s algorithm

- With this knowledge of union/find, how, exactly can it be used as a part of Kruskal’s algorithm?
  - What is the runtime of Kruskal’s algorithm?

From our Weighted Graphs Slides

- Kruskal's MST:
  - Insert all edges into a PQ
  - Grab the min edge from the PQ that does not create a cycle in the MST
  - Remove it from the PQ and add it to the MST