CS/COE 1501

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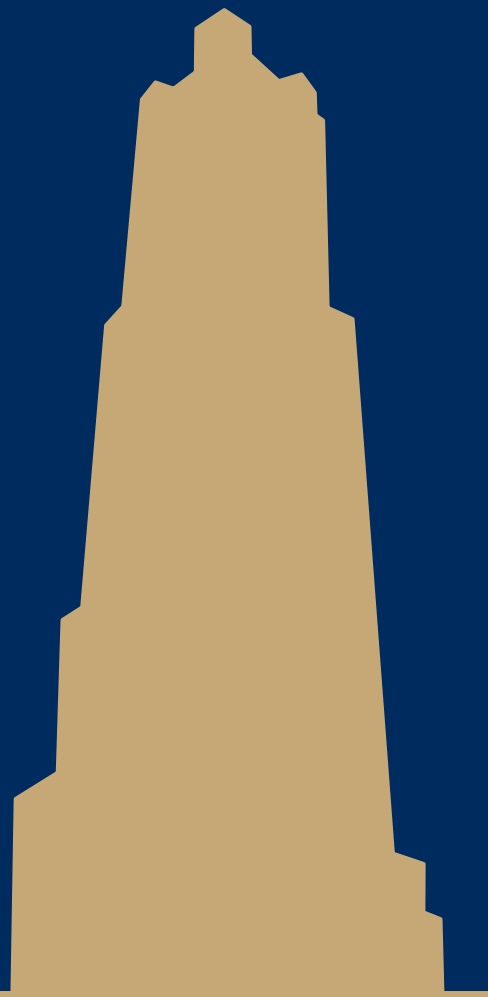
Priority Queues



We mentioned priority queues in building Huffman tries

- Primary operations they needed:
  - Insert
  - Find item with highest priority
    - E.g., findMin() or findMax()
  - Remove an item with highest priority
    - E.g., removeMin() or removeMax()
- How do we implement these operations?
  - Simplest approach: arrays

Unsorted array PQ

- Insert
- Find
- Remove
- Total runtime for n inserts and deletes?

Sorted array PQ

- Insert
- Find
- Remove
- Total runtime for n inserts and deletes?

So what other options do we have?

- What about a binary search tree?
  - Insert
  - Find
  - Remove

  - Average case vs. Worst case

Is a BST overkill?

- Our find and remove operations only need the highest priority item, not to find/remove *any* item
  - Can we take advantage of this to improve our runtime?

The heap

- A heap is complete binary tree such that for each node T in the tree:
  - T.val is of a higher priority than T.right_child.val
  - T.val is of a higher priority than T.left_child.val

- It does not matter how T.left_child.val relates to T.right_child.val
  - This is a relaxation of the approach needed by a BST

The *heap property*

Heap PQ runtimes

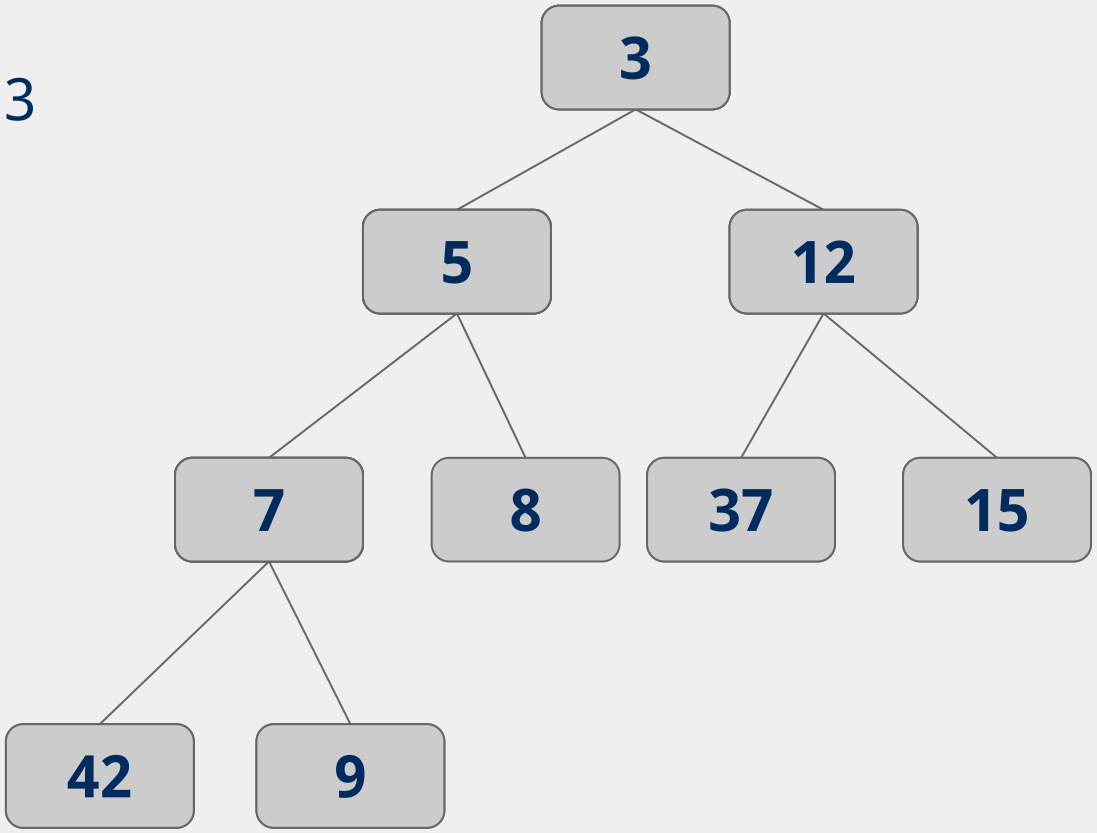
- Find is easy
  - Simply the root of the tree
    - $\Theta(1)$
- Remove and insert are not quite so trivial
  - The tree is modified and the heap property must be maintained

Heap insert

- Add a new node at the next available leaf
- Push the new node up the tree until it is supporting the heap property

Min heap insert

Insert:
7, 42, 37, 5, 8, 15, 12, 9, 3

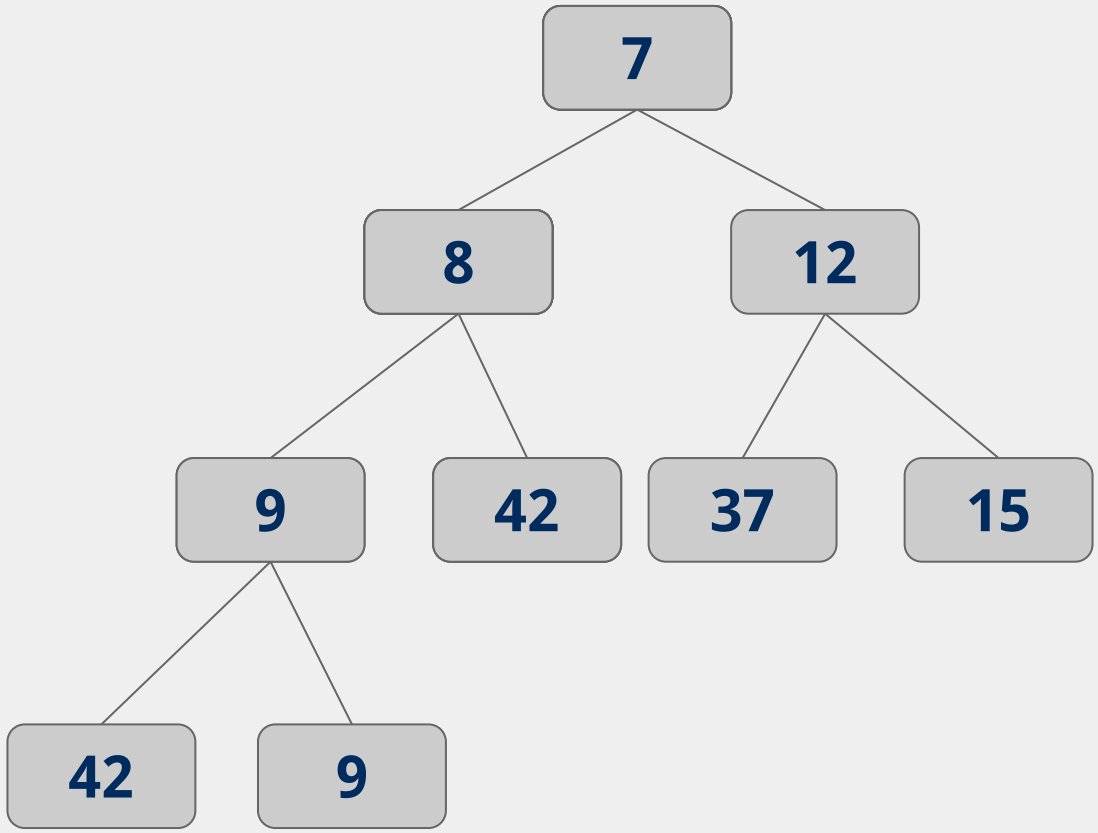


Heap remove

- Tricky to delete root…
  - So let's simply overwrite the root with the value of the last leaf and delete the last leaf
    - But then the root is violating the heap property…
      - So we push the root down the tree until it is supporting the heap property

Min heap removal

NO!



Heap runtimes

- Find
  - $\Theta(1)$
- Insert and remove
  - Height of a complete binary tree is lg n
  - At most, upheap and downheap operations traverse the height of the tree
  - Hence, insert and remove are $\Theta(\lg n)$

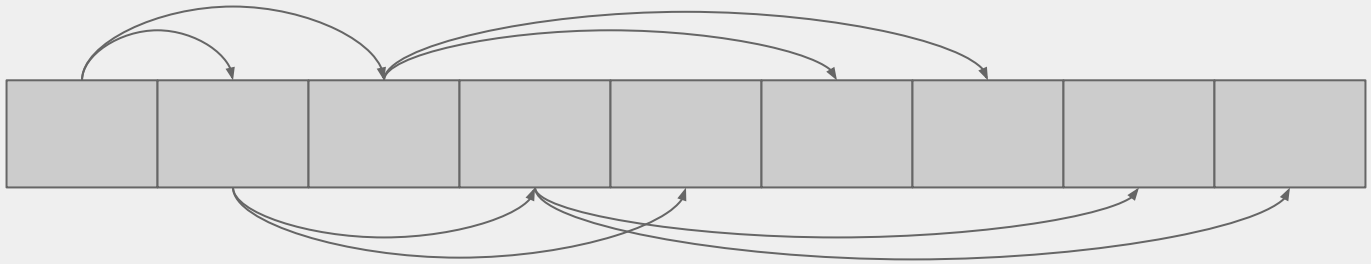
Heap implementation

- Simply implement tree nodes like for BST
  - This requires overhead for dynamic node allocation
  - Also must follow chains of parent/child relations to traverse the tree
- Note that a heap will be a complete binary tree…
  - We can easily represent a complete binary tree using an array

Storing a heap in an array

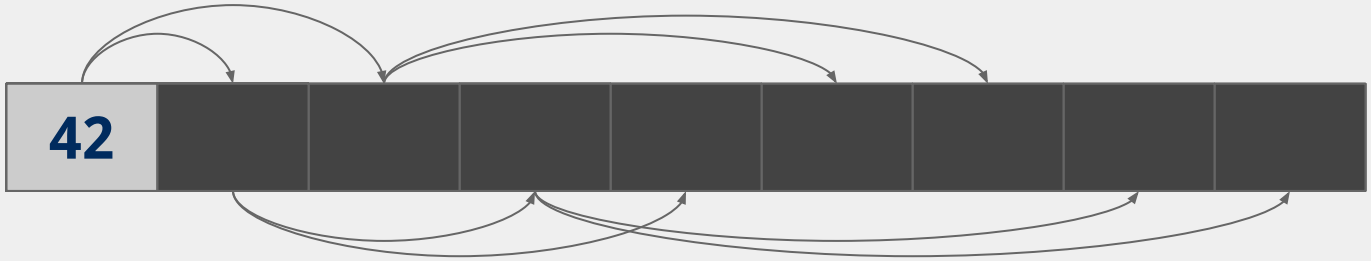
- Number nodes row-wise starting at 0
- Use these numbers as index values in the array
- Now, for node at index i
  - parent(i) = ⌊(i - 1) / 2⌋
  - left_child(i) = 2i + 1
  - right_child(i) = 2i + 2

For arrays indexed from 0



Heap Sort

- Heapify the numbers
  - MAX heap to sort ascending
  - MIN heap to sort descending
- “Remove” the root
  - Don’t actually delete the leaf node
- Consider the heap to be from 0 .. length - 1
- Repeat



Heap sort analysis

- Runtime:
  - Worst case:
    - n log n
- In-place?
  - Yes
- Stable?
  - No

Storing Objects in PQ

- What if we want to update an Object?
  - What is the runtime to find an arbitrary item in a heap?
    - Θ(n)
    - Hence, updating an item in the heap is Θ(n)
  - Can we improve of this?
    - Back the PQ with something other than a heap?
    - Develop a clever work around?

Indirection

- Maintain a second data structure that maps item IDs to each item's current position in the heap
- This creates an *indexable* PQ

Indirection example setup

- Let's say I'm shopping for a new video card and want to build a heap to help me keep track of the lowest price available from different stores.
- Keep objects of the following type in the heap:

```
class CardPrice implements Comparable<CardPrice>{
   public String store;
   public double price;
   public CardPrice(String s, double p) { … }
   public int compareTo(CardPrice o) {
      if (price < o.price) { return -1; }
      else if (price > o.price) { return 1; }
      else { return 0; }
   }
}
```

Indirection example

- n = new CardPrice("NE", 333.98);
- a = new CardPrice("AMZN", 339.99);
- x = new CardPrice("NCIX", 338.00);
- b = new CardPrice("BB", 349.99);

- Update price for NE: 340.00
- Update price for NCIX: 345.00
- Update price for BB: 200.00

Indirection

"NE":2
"AMZN":1
"NCIX":3
"BB":0